Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [119]. This is test number [34].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (119)	0.00 (0)
Mathematica	100.00 (119)	0.00 (0)
Maple	100.00 (119)	0.00 (0)
Mupad	100.00 (119)	0.00 (0)
Giac	89.92 (107)	10.08 (12)
Fricas	88.24 (105)	11.76 (14)
Sympy	57.98 (69)	% 42.02 (50)
Maxima	55.46 (66)	44.54 (53)
IntegrateAlgebraic	8.40 (10)	91.60 (109)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

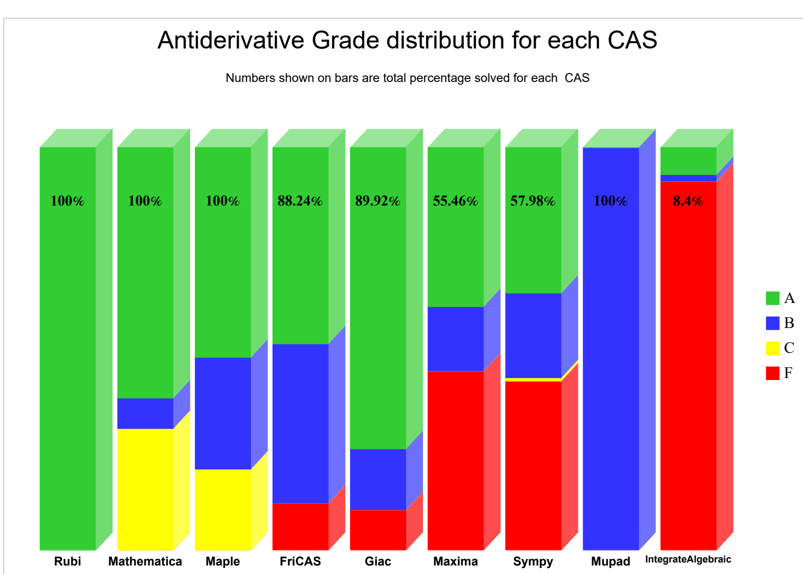
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

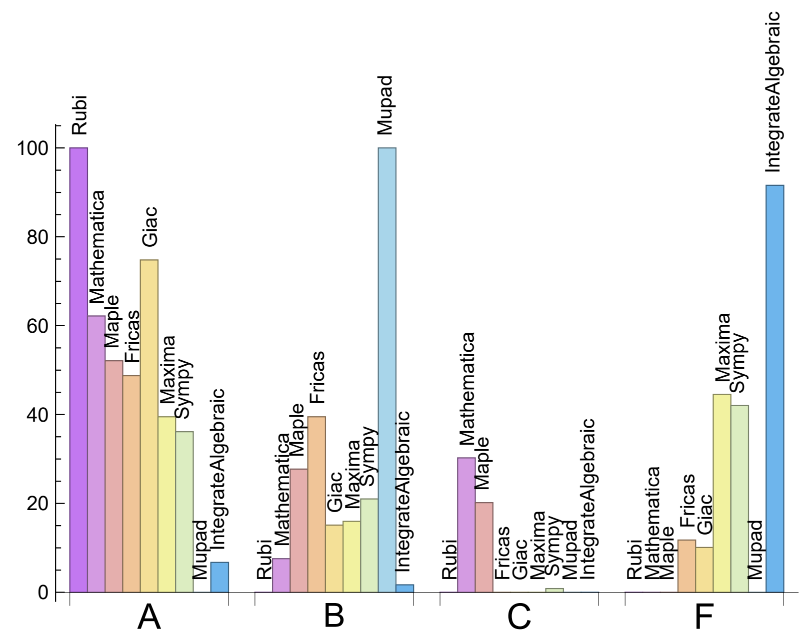
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Giac	74.79	15.13	0.00	10.08
Mathematica	62.18	7.56	30.25	0.00
Maple	52.10	27.73	20.17	0.00
Fricas	48.74	39.50	0.00	11.76
Maxima	39.50	15.97	0.00	44.54
Sympy	36.13	21.01	0.84	42.02
IntegrateAlgebraic	6.72	1.68	0.00	91.60
Mupad	N/A	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	14	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	109	100.00 %	0.00 %	0.00 %
Giac	12	41.67 %	41.67 %	16.67 %
Maxima	53	45.28 %	7.55 %	47.17 %
Sympy	50	0.00 %	100.00 %	0.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

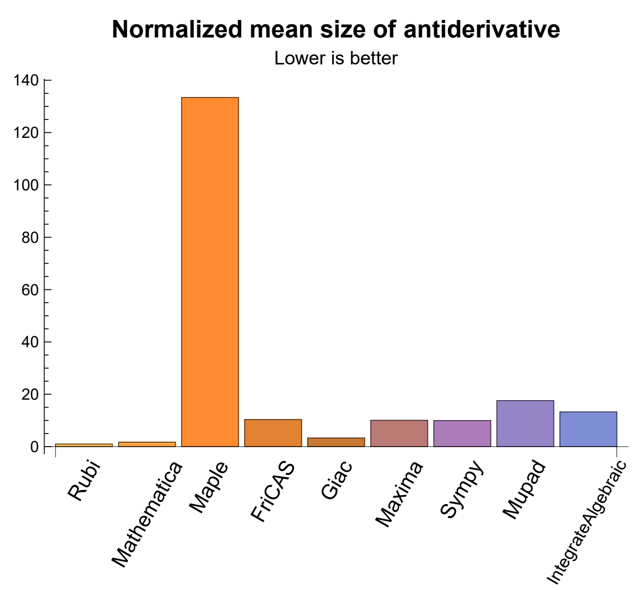
1.3 Performance

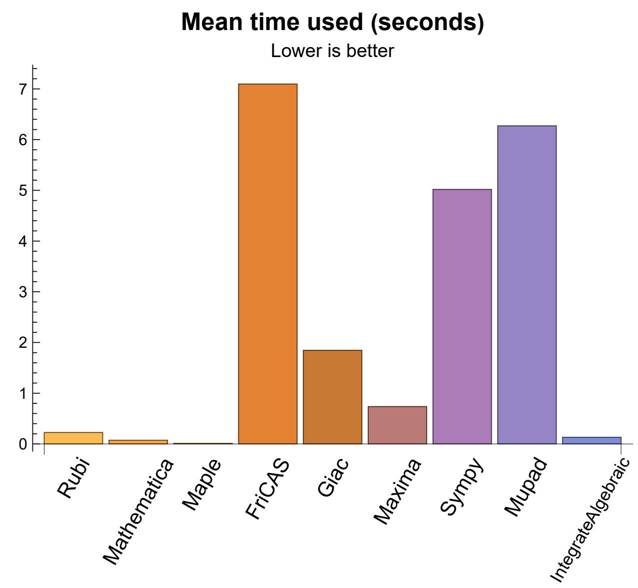
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	152.62	1.00	47.00	1.00
Mathematica	0.07	89.98	1.72	55.00	0.99
Maple	0.01	2548.18	133.39	51.00	1.04
Maxima	0.74	223.82	10.05	37.00	1.01
Fricas	7.10	967.82	10.36	218.00	2.54
Sympy	5.02	228.09	9.96	76.00	1.03
Giac	1.85	226.30	3.30	76.00	1.00
Mupad	6.27	3369.04	17.59	313.00	5.10
IntegrateAlgebraic	0.13	317.10	13.26	23.50	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

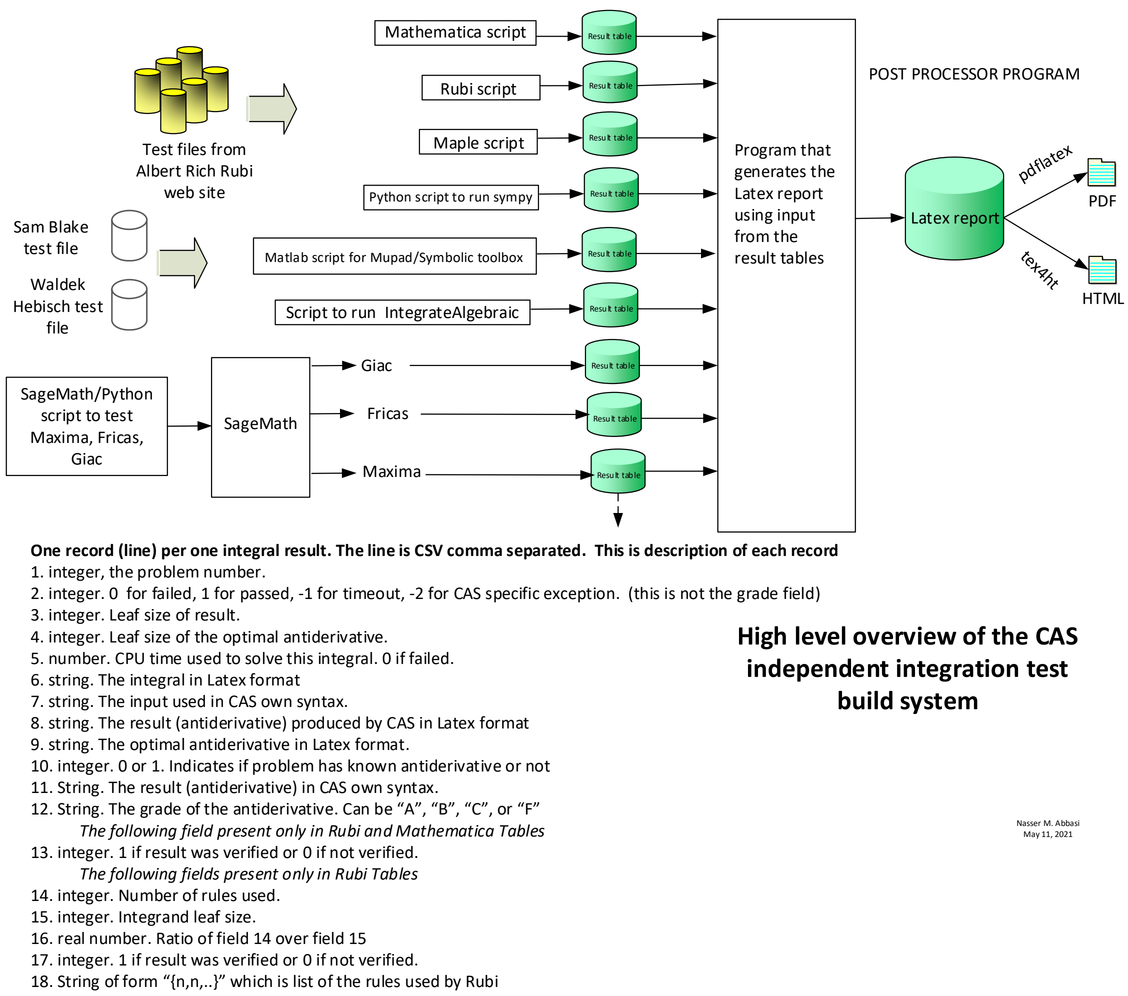
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 36, 38, 45, 47, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119 }

B grade: { 71, 72, 73, 75, 76, 77, 79, 80, 81 }

C grade: { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 48, 49, 50, 51, 52, 116, 117, 118 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 36, 40, 45, 47, 49, 51, 56, 57, 58, 59, 66, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119 }

B grade: { 6, 9, 38, 42, 53, 54, 55, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 103, 104, 105, 106 }

C grade: { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 35, 37, 39, 41, 43, 44, 46, 48, 50, 52, 118 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 32, 33, 45, 49, 71, 75, 79, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 99, 100, 101, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

B grade: { 72, 73, 74, 76, 77, 78, 80, 81, 82, 88, 89, 90, 96, 97, 98, 102, 104, 105, 106 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 36, 40, 44, 45, 47, 49, 50, 53, 54, 55, 56, 57, 58, 83, 84, 85, 86, 91, 92, 93, 94, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

B grade: { 8, 16, 17, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 42, 46, 48, 51, 52, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 95, 96, 97, 98, 103, 104, 105, 106 }

C grade: { }

F grade: { 14, 15, 18, 19, 35, 41, 43, 59, 60, 61, 62, 68, 69, 70 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 44, 45, 46, 47, 48, 49, 50, 51, 52, 83, 84, 85, 91, 92, 93, 99, 100, 101, 102, 115 }

B grade: { 9, 10, 11, 36, 71, 72, 73, 75, 76, 77, 79, 80, 81, 87, 88, 89, 95, 96, 97, 103, 104, 105, 107, 111, 116 }

C grade: { 119 }

F grade: { 12, 13, 14, 15, 16, 17, 18, 19, 35, 37, 38, 39, 40, 41, 42, 43, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 78, 82, 86, 90, 94, 98, 106, 108, 109, 110, 112, 113, 114, 117, 118 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 36, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { 25, 26, 27, 28, 29, 30, 31, 38, 42, 71, 72, 73, 74, 75, 76, 77, 78, 82 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 35, 37, 39, 41, 43, 119 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

C grade: { }

F grade: { }

2.1.9 IntegrateAlgebraic

A grade: { 82, 86, 90, 94, 98, 102, 106, 119 }

B grade: { 74, 78 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 79, 80, 81, 83, 84, 85, 87, 88, 89, 91, 92, 93, 95, 96, 97, 99, 100, 101, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	164	169	166	182	187	173	158	0
N.S.	1	1.00	1.01	1.04	1.02	1.12	1.15	1.06	0.97	0.00
time (sec)	N/A	0.185	0.047	0.002	0.813	1.073	0.101	0.340	1.599	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	135	136	135	147	151	141	130	0
N.S.	1	1.00	1.00	1.01	1.00	1.09	1.12	1.04	0.96	0.00
time (sec)	N/A	0.125	0.037	0.000	0.720	0.639	0.093	0.341	0.056	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	104	103	102	112	117	109	102	0
N.S.	1	1.00	1.01	1.00	0.99	1.09	1.14	1.06	0.99	0.00
time (sec)	N/A	0.097	0.029	0.000	0.653	0.865	0.085	0.304	0.043	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	70	69	76	75	76	70	0
N.S.	1	1.00	1.00	0.96	0.95	1.04	1.03	1.04	0.96	0.00
time (sec)	N/A	0.062	0.022	0.000	0.747	0.914	0.077	0.332	0.035	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	37	36	40	39	43	38	0
N.S.	1	1.00	1.00	0.88	0.86	0.95	0.93	1.02	0.90	0.00
time (sec)	N/A	0.028	0.009	0.001	0.552	1.028	0.066	0.325	0.043	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	176	313	169	465	175	173	165	0
N.S.	1	1.00	0.94	1.66	0.90	2.47	0.93	0.92	0.88	0.00
time (sec)	N/A	0.211	0.155	0.007	1.523	0.977	0.901	0.372	0.269	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	199	345	204	697	206	199	187	0
N.S.	1	1.00	0.93	1.62	0.96	3.27	0.97	0.93	0.88	0.00
time (sec)	N/A	0.226	0.192	0.008	1.600	1.084	1.676	0.380	1.801	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	209	362	240	941	246	224	221	0
N.S.	1	1.00	0.86	1.50	0.99	3.89	1.02	0.93	0.91	0.00
time (sec)	N/A	0.262	0.267	0.010	1.694	0.863	5.234	0.401	0.288	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	126	260	0	430	620	131	3586	0
N.S.	1	1.00	0.95	1.97	0.00	3.26	4.70	0.99	27.17	0.00
time (sec)	N/A	0.218	0.068	0.006	0.000	1.770	55.468	0.999	2.404	0.001

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	93	175	0	305	434	95	2624	0
N.S.	1	1.00	0.96	1.80	0.00	3.14	4.47	0.98	27.05	0.00
time (sec)	N/A	0.120	0.070	0.003	0.000	1.161	17.489	1.065	2.950	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	71	99	0	216	287	70	1632	0
N.S.	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	22.67	0.00
time (sec)	N/A	0.073	0.051	0.004	0.000	1.285	6.548	1.206	2.627	0.001

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	80	106	0	240	0	76	4149	0
N.S.	1	1.00	1.03	1.36	0.00	3.08	0.00	0.97	53.19	0.00
time (sec)	N/A	0.128	0.035	0.006	0.000	1.356	0.000	1.046	6.765	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	130	191	0	385	0	128	7282	0
N.S.	1	1.00	1.16	1.71	0.00	3.44	0.00	1.14	65.02	0.00
time (sec)	N/A	0.197	0.050	0.008	0.000	2.597	0.000	1.071	9.569	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	723	723	88	70	0	0	0	0	13112	0
N.S.	1	1.00	0.12	0.10	0.00	0.00	0.00	0.00	18.14	0.00
time (sec)	N/A	1.813	0.048	0.013	0.000	0.000	0.000	0.000	42.007	0.001

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	718	718	88	67	0	0	0	0	11453	0
N.S.	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	15.95	0.00
time (sec)	N/A	1.457	0.050	0.004	0.000	0.000	0.000	0.000	30.152	0.001

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	634	634	59	49	0	13607	0	0	7457	0
N.S.	1	1.00	0.09	0.08	0.00	21.46	0.00	0.00	11.76	0.00
time (sec)	N/A	0.728	0.030	0.005	0.000	119.716	0.000	0.000	24.559	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	634	634	61	47	0	14094	0	0	7469	0
N.S.	1	1.00	0.10	0.07	0.00	22.23	0.00	0.00	11.78	0.00
time (sec)	N/A	0.654	0.030	0.007	0.000	39.394	0.000	0.000	18.962	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	653	653	85	70	0	0	0	0	11174	0
N.S.	1	1.00	0.13	0.11	0.00	0.00	0.00	0.00	17.11	0.00
time (sec)	N/A	1.175	0.047	0.013	0.000	0.000	0.000	0.000	38.020	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	655	655	89	68	0	0	0	0	13466	0
N.S.	1	1.00	0.14	0.10	0.00	0.00	0.00	0.00	20.56	0.00
time (sec)	N/A	1.110	0.047	0.010	0.000	0.000	0.000	0.000	37.903	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	38	37	37	42	37	39	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85	0.00
time (sec)	N/A	0.058	0.016	0.003	0.988	1.460	0.137	0.418	0.056	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	25	24	24	32	24	26	0
N.S.	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84	0.00
time (sec)	N/A	0.035	0.008	0.003	0.960	1.639	0.121	0.577	0.038	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.040	0.009	0.003	0.951	1.353	0.135	0.568	0.046	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	44	35	38	34	41	35	36	0
N.S.	1	1.00	1.07	0.85	0.93	0.83	1.00	0.85	0.88	0.00
time (sec)	N/A	0.055	0.013	0.006	0.973	1.457	0.149	0.592	1.859	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	45	25	24	28	36	24	26	0
N.S.	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84	0.00
time (sec)	N/A	0.045	0.013	0.006	0.960	1.063	0.144	0.449	0.045	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	418	418	47	46	0	1036	31	642	332	0
N.S.	1	1.00	0.11	0.11	0.00	2.48	0.07	1.54	0.79	0.00
time (sec)	N/A	0.538	0.011	0.007	0.000	1.328	0.182	0.576	0.652	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	382	382	48	44	0	1588	32	817	309	0
N.S.	1	1.00	0.13	0.12	0.00	4.16	0.08	2.14	0.81	0.00
time (sec)	N/A	0.328	0.013	0.004	0.000	1.471	0.185	0.692	2.279	0.001

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	378	378	46	41	0	1030	24	632	330	0
N.S.	1	1.00	0.12	0.11	0.00	2.72	0.06	1.67	0.87	0.00
time (sec)	N/A	0.259	0.013	0.006	0.000	1.386	0.180	0.626	2.376	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	55	44	0	1583	22	821	281	0
N.S.	1	1.00	0.13	0.11	0.00	3.85	0.05	2.00	0.68	0.00
time (sec)	N/A	0.276	0.013	0.005	0.000	1.521	0.181	0.582	2.264	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	57	44	0	1031	26	637	319	0
N.S.	1	1.00	0.14	0.11	0.00	2.51	0.06	1.55	0.78	0.00
time (sec)	N/A	0.277	0.012	0.004	0.000	1.270	0.179	0.722	2.299	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	47	46	0	1598	31	829	313	0
N.S.	1	1.00	0.11	0.11	0.00	3.84	0.07	1.99	0.75	0.00
time (sec)	N/A	0.275	0.014	0.007	0.000	1.512	0.195	0.708	0.398	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	418	418	47	46	0	1062	32	642	332	0
N.S.	1	1.00	0.11	0.11	0.00	2.54	0.08	1.54	0.79	0.00
time (sec)	N/A	0.359	0.012	0.007	0.000	1.347	0.200	0.642	2.399	0.001

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	37	33	32	32	37	32	34	0
N.S.	1	1.00	1.03	0.92	0.89	0.89	1.03	0.89	0.94	0.00
time (sec)	N/A	0.039	0.010	0.003	0.977	1.093	0.134	0.462	1.845	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	55	35	38	34	41	35	36	0
N.S.	1	1.00	1.41	0.90	0.97	0.87	1.05	0.90	0.92	0.00
time (sec)	N/A	0.056	0.014	0.006	0.989	1.100	0.150	0.540	1.847	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	55	35	0	34	41	35	36	0
N.S.	1	1.00	1.41	0.90	0.00	0.87	1.05	0.90	0.92	0.00
time (sec)	N/A	0.063	0.010	0.006	0.000	1.576	0.144	0.410	0.039	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	433	433	88	67	0	0	0	0	50213	0
N.S.	1	1.00	0.20	0.15	0.00	0.00	0.00	0.00	115.97	0.00
time (sec)	N/A	1.132	0.075	0.006	0.000	0.000	0.000	0.000	9.632	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	71	99	0	216	287	70	3704	0
N.S.	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	51.44	0.00
time (sec)	N/A	0.072	0.057	0.004	0.000	1.105	18.295	20.738	4.206	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	59	51	0	13521	0	0	29445	0
N.S.	1	1.00	0.16	0.14	0.00	36.06	0.00	0.00	78.52	0.00
time (sec)	N/A	0.456	0.046	0.003	0.000	47.545	0.000	0.000	9.569	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	179	340	0	1535	0	1406	4501	0
N.S.	1	1.00	0.97	1.85	0.00	8.34	0.00	7.64	24.46	0.00
time (sec)	N/A	0.213	0.152	0.020	0.000	1.322	0.000	20.309	7.053	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	61	47	0	13304	0	0	36707	0
N.S.	1	1.00	0.16	0.13	0.00	35.48	0.00	0.00	97.89	0.00
time (sec)	N/A	0.351	0.047	0.002	0.000	10.109	0.000	0.000	8.746	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	80	106	0	240	0	78	8454	0
N.S.	1	1.00	1.03	1.36	0.00	3.08	0.00	1.00	108.38	0.00
time (sec)	N/A	0.126	0.033	0.007	0.000	2.511	0.000	20.628	5.271	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	392	392	85	72	0	0	0	0	39028	0
N.S.	1	1.00	0.22	0.18	0.00	0.00	0.00	0.00	99.56	0.00
time (sec)	N/A	0.683	0.064	0.007	0.000	0.000	0.000	0.000	9.459	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	89	365	0	2772	0	3006	15013	0
N.S.	1	1.00	0.45	1.83	0.00	13.93	0.00	15.11	75.44	0.00
time (sec)	N/A	0.311	0.045	0.023	0.000	2.211	0.000	22.519	7.620	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	86	68	0	0	0	0	65350	0
N.S.	1	1.00	0.22	0.17	0.00	0.00	0.00	0.00	165.86	0.00
time (sec)	N/A	0.625	0.071	0.007	0.000	0.000	0.000	0.000	10.224	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	46	34	0	218	170	208	56	0
N.S.	1	1.00	0.17	0.12	0.00	0.78	0.61	0.75	0.20	0.00
time (sec)	N/A	0.300	0.016	0.007	0.000	1.270	0.226	0.452	1.920	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.042	0.014	0.006	1.507	0.731	0.146	0.628	0.049	0.001

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	55	46	0	715	27	253	248	0
N.S.	1	1.00	0.15	0.13	0.00	2.01	0.08	0.71	0.70	0.00
time (sec)	N/A	0.289	0.016	0.008	0.000	1.108	3.164	0.481	1.985	0.001

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	44	39	0	41	42	31	20	0
N.S.	1	1.00	0.88	0.78	0.00	0.82	0.84	0.62	0.40	0.00
time (sec)	N/A	0.040	0.016	0.017	0.000	0.868	0.129	0.440	1.889	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	57	44	0	715	26	253	208	0
N.S.	1	1.00	0.16	0.12	0.00	2.01	0.07	0.71	0.59	0.00
time (sec)	N/A	0.216	0.014	0.000	0.000	0.689	3.226	0.536	0.002	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	44	35	38	34	41	38	36	0
N.S.	1	1.00	1.07	0.85	0.93	0.83	1.00	0.93	0.88	0.00
time (sec)	N/A	0.053	0.013	0.007	0.965	0.890	0.160	0.453	1.886	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	47	38	0	224	168	210	58	0
N.S.	1	1.00	0.17	0.14	0.00	0.80	0.60	0.75	0.21	0.00
time (sec)	N/A	0.208	0.015	0.010	0.000	0.990	0.234	0.582	1.860	0.001

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	49	70	0	188	76	81	56	0
N.S.	1	1.00	0.55	0.79	0.00	2.11	0.85	0.91	0.63	0.00
time (sec)	N/A	0.090	0.016	0.011	0.000	1.032	0.232	0.528	0.099	0.001

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	47	46	0	608	32	258	479	0
N.S.	1	1.00	0.13	0.12	0.00	1.64	0.09	0.70	1.29	0.00
time (sec)	N/A	0.269	0.015	0.009	0.000	1.188	3.175	0.444	0.068	0.001

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	283	662	0	1027	0	295	2490	0
N.S.	1	1.00	1.01	2.36	0.00	3.67	0.00	1.05	8.89	0.00
time (sec)	N/A	0.597	0.229	0.014	0.000	94.895	0.000	0.376	6.206	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	218	512	0	798	0	224	2051	0
N.S.	1	1.00	1.00	2.35	0.00	3.66	0.00	1.03	9.41	0.00
time (sec)	N/A	0.395	0.172	0.010	0.000	52.349	0.000	0.368	5.242	0.001

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	178	388	0	596	0	185	1367	0
N.S.	1	1.00	1.01	2.20	0.00	3.39	0.00	1.05	7.77	0.00
time (sec)	N/A	0.285	0.185	0.007	0.000	16.037	0.000	0.401	4.339	0.001

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	132	275	0	405	0	149	966	0
N.S.	1	1.00	0.89	1.85	0.00	2.72	0.00	1.00	6.48	0.00
time (sec)	N/A	0.210	0.119	0.006	0.000	5.760	0.000	0.366	3.668	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	107	169	0	305	0	127	801	0
N.S.	1	1.00	0.86	1.36	0.00	2.46	0.00	1.02	6.46	0.00
time (sec)	N/A	0.145	0.074	0.006	0.000	2.168	0.000	0.392	3.407	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	105	168	0	305	0	126	521	0
N.S.	1	1.00	0.85	1.37	0.00	2.48	0.00	1.02	4.24	0.00
time (sec)	N/A	0.107	0.073	0.006	0.000	2.054	0.000	0.341	3.817	0.001

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	159	152	285	0	0	0	164	2399	0
N.S.	1	1.01	0.96	1.80	0.00	0.00	0.00	1.04	15.18	0.00
time (sec)	N/A	0.271	0.186	0.007	0.000	0.000	0.000	0.351	5.400	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	194	412	0	0	0	210	2388	0
N.S.	1	1.00	1.01	2.13	0.00	0.00	0.00	1.09	12.37	0.00
time (sec)	N/A	0.343	0.174	0.011	0.000	0.000	0.000	0.342	20.389	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	252	562	0	0	0	279	3530	0
N.S.	1	1.00	1.00	2.23	0.00	0.00	0.00	1.11	14.01	0.00
time (sec)	N/A	0.428	0.220	0.013	0.000	0.000	0.000	0.348	26.162	0.001

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	338	943	0	0	0	565	3503	0
N.S.	1	1.00	0.99	2.75	0.00	0.00	0.00	1.65	10.21	0.00
time (sec)	N/A	0.907	0.356	0.014	0.000	0.000	0.000	0.420	8.039	0.001

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	274	274	269	765	0	2139	0	476	2495	0
N.S.	1	1.00	0.98	2.79	0.00	7.81	0.00	1.74	9.11	0.00
time (sec)	N/A	0.563	0.287	0.011	0.000	158.655	0.000	0.401	6.003	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	207	580	0	1465	0	412	2037	0
N.S.	1	1.00	0.84	2.36	0.00	5.96	0.00	1.67	8.28	0.00
time (sec)	N/A	0.395	0.227	0.008	0.000	56.306	0.000	0.422	5.112	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	159	389	0	1120	0	331	1585	0
N.S.	1	1.00	0.82	2.01	0.00	5.77	0.00	1.71	8.17	0.00
time (sec)	N/A	0.306	0.225	0.010	0.000	19.721	0.000	0.347	6.091	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	148	328	0	1059	0	323	1768	0
N.S.	1	1.00	0.81	1.79	0.00	5.79	0.00	1.77	9.66	0.00
time (sec)	N/A	0.236	0.244	0.008	0.000	16.697	0.000	0.366	8.070	0.001

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	151	386	0	1079	0	331	1782	0
N.S.	1	1.00	0.80	2.04	0.00	5.71	0.00	1.75	9.43	0.00
time (sec)	N/A	0.305	0.211	0.009	0.000	9.280	0.000	0.354	8.109	0.001

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	249	246	589	0	0	0	391	3510	0
N.S.	1	1.00	0.99	2.38	0.00	0.00	0.00	1.58	14.15	0.00
time (sec)	N/A	0.409	0.253	0.013	0.000	0.000	0.000	0.408	25.284	0.001

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	287	791	0	0	0	487	4948	0
N.S.	1	1.00	0.99	2.72	0.00	0.00	0.00	1.67	17.00	0.00
time (sec)	N/A	0.563	0.340	0.014	0.000	0.000	0.000	0.361	31.159	0.002

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	372	372	370	993	0	0	0	587	7144	0
N.S.	1	1.00	0.99	2.67	0.00	0.00	0.00	1.58	19.20	0.00
time (sec)	N/A	0.851	0.425	0.016	0.000	0.000	0.000	0.488	45.611	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	201	46548	14	1446	1326	216	1203	0
N.S.	1	1.00	12.56	2909.25	0.88	90.38	82.88	13.50	75.19	0.00
time (sec)	N/A	0.060	0.176	0.003	0.430	0.762	0.350	0.434	3.341	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	233	46552	1240	1454	1384	246	1210	0
N.S.	1	1.00	12.94	2586.22	68.89	80.78	76.89	13.67	67.22	0.00
time (sec)	N/A	0.329	0.177	0.003	0.490	0.763	0.338	0.661	3.234	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	233	46552	1240	1454	1394	246	1210	0
N.S.	1	1.00	12.94	2586.22	68.89	80.78	77.44	13.67	67.22	0.00
time (sec)	N/A	0.302	0.182	0.003	0.546	0.773	0.342	0.609	3.183	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	2042	2041	1297	0	1693	1395	1485
N.S.	1	1.00	0.96	88.78	88.74	56.39	0.00	73.61	60.65	64.57
time (sec)	N/A	0.056	0.068	0.062	0.857	1.218	0.000	1.002	5.778	0.350

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	201	47685	16	1450	1326	218	1208	0
N.S.	1	1.00	11.17	2649.17	0.89	80.56	73.67	12.11	67.11	0.00
time (sec)	N/A	0.069	0.175	0.004	0.444	0.889	0.360	0.473	1.376	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	233	47688	1242	1454	1384	246	1214	0
N.S.	1	1.00	11.65	2384.40	62.10	72.70	69.20	12.30	60.70	0.00
time (sec)	N/A	0.322	0.169	0.002	0.525	0.814	0.359	0.572	3.251	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	233	47688	1242	1454	1394	246	1214	0
N.S.	1	1.00	11.65	2384.40	62.10	72.70	69.70	12.30	60.70	0.00
time (sec)	N/A	0.310	0.167	0.001	0.505	0.804	0.360	0.683	1.282	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	2046	2045	1299	0	1693	1401	1485
N.S.	1	1.00	0.96	81.84	81.80	51.96	0.00	67.72	56.04	59.40
time (sec)	N/A	0.060	0.054	0.059	0.827	0.861	0.000	1.089	5.776	0.383

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	172	155	13	154	175	13	154	0
N.S.	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27	0.00
time (sec)	N/A	0.014	0.005	0.001	0.435	0.551	0.129	0.402	2.088	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	182	157	156	156	182	15	156	0
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	0.94	9.75	0.00
time (sec)	N/A	0.054	0.006	0.001	0.433	0.724	0.127	0.383	2.080	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	186	157	156	156	185	15	156	0
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	0.94	9.75	0.00
time (sec)	N/A	0.056	0.006	0.001	0.439	0.741	0.127	0.508	2.079	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	230	229	189	0	189	229	21
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90	1.00
time (sec)	N/A	0.033	0.117	0.035	0.478	0.866	0.000	0.433	2.628	0.058

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	10	12	11	11	10	12	11	0
N.S.	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00	0.00
time (sec)	N/A	0.004	0.003	0.002	0.446	1.064	0.155	0.347	1.962	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	16	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88	0.00
time (sec)	N/A	0.019	0.006	0.002	0.428	1.037	0.280	1.734	1.956	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	16	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88	0.00
time (sec)	N/A	0.024	0.006	0.001	0.439	0.807	0.413	1.092	0.052	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	24	23	19	0	19	121	19
N.S.	1	1.00	1.00	1.26	1.21	1.00	0.00	1.00	6.37	1.00
time (sec)	N/A	0.027	0.105	0.025	0.598	1.130	0.000	0.464	2.317	0.050

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	15	15	14	350	359	14	358	0
N.S.	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38	0.00
time (sec)	N/A	0.005	0.012	0.000	0.438	0.655	4.791	0.395	3.616	0.001

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	352	352	360	16	360	0
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00	0.00
time (sec)	N/A	0.020	0.012	0.000	0.948	1.052	7.662	6.776	12.162	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	352	352	360	16	360	0
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00	0.00
time (sec)	N/A	0.023	0.013	0.002	0.954	1.012	11.757	22.371	18.211	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	22	416	394	0	21	496	23
N.S.	1	1.00	0.96	0.96	18.09	17.13	0.00	0.91	21.57	1.00
time (sec)	N/A	0.027	0.060	0.064	2.393	1.064	0.000	0.633	23.011	0.072

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	12	14	13	13	10	14	13	0
N.S.	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00	0.00
time (sec)	N/A	0.005	0.005	0.001	0.435	0.848	0.151	0.390	0.049	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	14	18	17	0
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89	0.00
time (sec)	N/A	0.019	0.007	0.000	0.437	0.632	0.281	1.624	0.049	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	14	18	17	0
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89	0.00
time (sec)	N/A	0.024	0.007	0.001	0.431	0.764	0.392	1.040	0.059	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	26	25	21	0	21	199	21
N.S.	1	1.00	1.00	1.24	1.19	1.00	0.00	1.00	9.48	1.00
time (sec)	N/A	0.029	0.117	0.025	0.599	1.219	0.000	0.374	2.676	0.080

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	16	17	16	354	359	16	358	0
N.S.	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89	0.00
time (sec)	N/A	0.004	0.013	0.002	0.428	0.982	5.060	0.427	5.221	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	356	356	360	18	360	0
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00	0.00
time (sec)	N/A	0.020	0.017	0.000	0.967	0.999	8.089	7.187	11.044	0.002

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	356	356	360	18	360	0
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00	0.00
time (sec)	N/A	0.024	0.017	0.001	0.938	0.758	11.792	22.351	16.597	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	24	419	397	0	23	496	25
N.S.	1	1.00	0.92	0.96	16.76	15.88	0.00	0.92	19.84	1.00
time (sec)	N/A	0.029	0.066	0.067	2.423	0.974	0.000	0.790	22.399	0.081

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	9	9	10	10	8	11	8	0
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80	0.00
time (sec)	N/A	0.004	0.004	0.002	0.440	0.812	0.122	0.456	0.051	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	15	15	14	17	13	12	15	13	0
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81	0.00
time (sec)	N/A	0.024	0.006	0.006	0.426	0.612	0.191	0.486	0.064	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	15	15	14	17	13	12	15	13	0
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81	0.00
time (sec)	N/A	0.030	0.007	0.006	0.427	0.871	0.202	0.340	1.991	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	47	17	48	17	28	24
N.S.	1	1.00	1.00	1.20	3.13	1.13	3.20	1.13	1.87	1.60
time (sec)	N/A	0.035	0.012	0.022	0.442	0.863	31.234	0.374	2.225	0.058

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	14	177	13	81	87	13	12	0
N.S.	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80	0.00
time (sec)	N/A	0.004	0.020	0.016	0.422	0.857	0.879	0.318	4.296	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.021	0.029	0.019	0.520	0.885	1.384	0.450	2.332	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.025	0.037	0.013	0.514	0.713	1.897	0.606	5.065	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	203	612	105	0	20	107	21
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10	1.00
time (sec)	N/A	0.032	0.181	0.052	0.663	0.913	0.000	0.440	2.357	0.110

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	19	21	20	28	104	20	39	0
N.S.	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95	0.00
time (sec)	N/A	0.005	0.009	0.003	0.425	0.794	57.114	0.414	2.037	0.029

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	33	33	0	23	49	0
N.S.	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96	0.00
time (sec)	N/A	0.019	0.011	0.004	0.595	0.783	0.000	0.460	2.092	0.060

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	33	33	0	23	49	0
N.S.	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96	0.00
time (sec)	N/A	0.024	0.012	0.006	0.578	0.807	0.000	0.319	2.115	0.077

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	40	39	38	0	27	56	0
N.S.	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07	0.00
time (sec)	N/A	0.028	0.029	0.059	0.722	0.965	0.000	0.845	2.569	0.074

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	23	22	32	104	22	42	0
N.S.	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91	0.00
time (sec)	N/A	0.005	0.011	0.001	0.422	0.824	56.660	0.397	2.049	0.028

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	26	37	37	0	25	52	0
N.S.	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93	0.00
time (sec)	N/A	0.020	0.015	0.003	0.597	0.834	0.000	0.454	2.047	0.054

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	26	37	37	0	25	52	0
N.S.	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93	0.00
time (sec)	N/A	0.025	0.016	0.006	0.588	0.877	0.000	0.385	2.079	0.080

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	28	45	43	42	0	29	59	0
N.S.	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03	0.00
time (sec)	N/A	0.028	0.034	0.060	0.708	0.876	0.000	0.820	2.536	0.076

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	17	24	19	26	46	19	23	0
N.S.	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21	0.00
time (sec)	N/A	0.004	0.010	0.005	0.428	1.002	0.662	0.410	2.034	0.029

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	97	31	35	31	85	22	31	0
N.S.	1	1.00	4.04	1.29	1.46	1.29	3.54	0.92	1.29	0.00
time (sec)	N/A	0.014	0.073	0.003	0.588	0.847	17.117	0.400	2.074	0.105

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	97	31	35	31	0	22	31	0
N.S.	1	1.00	4.04	1.29	1.46	1.29	0.00	0.92	1.29	0.00
time (sec)	N/A	0.020	0.074	0.003	0.590	0.766	0.000	0.671	2.069	0.222

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	111	155	40	36	0	26	34	0
N.S.	1	1.00	4.27	5.96	1.54	1.38	0.00	1.00	1.31	0.00
time (sec)	N/A	0.079	0.130	0.106	0.766	0.881	0.000	0.834	2.126	0.134

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	36	34	33	126	0	31	47
N.S.	1	1.00	1.00	0.77	0.72	0.70	2.68	0.00	0.66	1.00
time (sec)	N/A	0.062	0.023	0.002	0.445	0.764	6.321	0.000	2.463	0.074

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [25] had the largest ratio of [.3913]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	22	0.045
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	22	0.045
5	A	2	1	1.00	20	0.050
6	A	8	8	1.00	22	0.364
7	A	8	8	1.00	22	0.364
8	A	8	8	1.00	22	0.364
9	A	7	6	1.00	25	0.240
10	A	6	6	1.00	25	0.240
11	A	5	5	1.00	25	0.200
12	A	7	6	1.00	25	0.240
13	A	7	6	1.00	25	0.240
14	A	14	8	1.00	25	0.320
15	A	14	8	1.00	25	0.320
16	A	13	7	1.00	23	0.304
17	A	13	7	1.00	22	0.318
18	A	14	8	1.00	25	0.320
19	A	14	8	1.00	25	0.320
20	A	7	6	1.00	23	0.261
21	A	4	4	1.00	23	0.174
22	A	5	5	1.00	23	0.217
23	A	7	6	1.00	23	0.261
24	A	5	4	1.00	23	0.174
25	A	15	9	1.00	23	0.391
26	A	15	9	1.00	23	0.391
27	A	14	8	1.00	23	0.348
28	A	13	7	1.00	21	0.333
29	A	13	7	1.00	20	0.350
30	A	14	8	1.00	23	0.348
31	A	15	9	1.00	23	0.391
32	A	5	5	1.00	21	0.238
33	A	7	6	1.00	21	0.286
34	A	8	7	1.00	18	0.389
35	A	8	5	1.00	25	0.200
36	A	5	5	1.00	25	0.200
37	A	7	4	1.00	25	0.160
38	A	4	3	1.00	23	0.130
39	A	7	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	7	6	1.00	25	0.240
41	A	8	5	1.00	25	0.200
42	A	5	4	1.00	25	0.160
43	A	8	5	1.00	25	0.200
44	A	20	7	1.00	23	0.304
45	A	5	5	1.00	23	0.217
46	A	21	7	1.00	23	0.304
47	A	4	3	1.00	21	0.143
48	A	19	6	1.00	20	0.300
49	A	7	6	1.00	23	0.261
50	A	20	7	1.00	23	0.304
51	A	11	8	1.00	23	0.348
52	A	21	9	1.00	23	0.391
53	A	7	6	1.00	25	0.240
54	A	7	6	1.00	25	0.240
55	A	7	6	1.00	23	0.261
56	A	7	6	1.00	22	0.273
57	A	7	6	1.00	25	0.240
58	A	7	7	1.00	25	0.280
59	A	7	6	1.01	25	0.240
60	A	7	6	1.00	25	0.240
61	A	7	6	1.00	25	0.240
62	A	7	6	1.00	25	0.240
63	A	7	6	1.00	25	0.240
64	A	7	6	1.00	23	0.261
65	A	7	6	1.00	22	0.273
66	A	7	6	1.00	25	0.240
67	A	8	7	1.00	25	0.280
68	A	7	6	1.00	25	0.240
69	A	7	6	1.00	25	0.240
70	A	7	6	1.00	25	0.240
71	A	1	1	1.00	19	0.053
72	A	2	2	1.00	24	0.083
73	A	2	2	1.00	26	0.077
74	A	2	2	1.00	30	0.067
75	A	1	1	1.00	21	0.048
76	A	2	2	1.00	26	0.077
77	A	2	2	1.00	28	0.071
78	A	2	2	1.00	32	0.062
79	A	1	1	1.00	18	0.056
80	A	3	3	1.00	23	0.130
81	A	3	3	1.00	25	0.120
82	A	3	3	1.00	29	0.103
83	A	1	1	1.00	19	0.053
84	A	2	2	1.00	24	0.083
85	A	2	2	1.00	26	0.077
86	A	2	2	1.00	30	0.067
87	A	1	1	1.00	19	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	2	2	1.00	24	0.083
89	A	2	2	1.00	26	0.077
90	A	2	2	1.00	30	0.067
91	A	1	1	1.00	21	0.048
92	A	2	2	1.00	26	0.077
93	A	2	2	1.00	28	0.071
94	A	2	2	1.00	32	0.062
95	A	1	1	1.00	21	0.048
96	A	2	2	1.00	26	0.077
97	A	2	2	1.00	28	0.071
98	A	2	2	1.00	32	0.062
99	A	1	1	1.00	18	0.056
100	A	4	3	0.94	23	0.130
101	A	4	3	0.94	25	0.120
102	A	4	3	1.00	29	0.103
103	A	1	1	1.00	18	0.056
104	A	3	3	1.00	23	0.130
105	A	3	3	1.00	25	0.120
106	A	3	3	1.00	29	0.103
107	A	1	1	1.00	19	0.053
108	A	2	2	1.00	24	0.083
109	A	2	2	1.00	26	0.077
110	A	2	2	1.00	30	0.067
111	A	1	1	1.00	21	0.048
112	A	2	2	1.00	26	0.077
113	A	2	2	1.00	28	0.071
114	A	2	2	1.00	32	0.062
115	A	1	1	1.00	18	0.056
116	A	1	1	1.00	23	0.043
117	A	1	1	1.00	25	0.040
118	A	2	2	1.00	29	0.069
119	A	3	3	1.00	59	0.051

Chapter 3

Listing of integrals

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3.1 $\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=163

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+\right.$$

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) \, dx &= \int \left(ad^5+d^4(bd+5ae)x^3+d^3\left(cd^2+5e(bd+2ae)\right)x^6+5d^2e\left(cd^2+2e(bd\right.\right.\\ &= ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7+\frac{1}{2}d^2e\left(cd^2+2e(bc\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 164, normalized size = 1.01

$$\frac{5}{13}de^2x^{13}\left(ae^2+2bde+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2ae^2+2bde+cd^2\right)+\frac{1}{16}e^3x^{16}\left(ae^2+5bde+10cd^2\right)+\frac{1}{7}d^3x^7\left(10ae^2+5bde+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7
)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*
d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)^5*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.07, size = 182, normalized size = 1.12

$$\frac{1}{22}x^{22}e^{5}c+\frac{5}{19}x^{19}e^{4}dc+\frac{1}{19}x^{19}e^{5}b+\frac{5}{8}x^{16}e^{3}d^{2}c+\frac{5}{16}x^{16}e^{4}db+\frac{1}{16}x^{16}e^{5}a+\frac{10}{13}x^{13}e^{2}d^{3}c+\frac{10}{13}x^{13}e^{3}d^{2}b+\frac{5}{13}x^{13}e^{4}da+\frac{1}{2}x^{10}ed^{4}c+x^{10}e^{2}d^{3}b+x^{10}e^{3}d^{2}a+\frac{1}{7}x^{7}d^{5}c+\frac{5}{7}x^{7}ed^{4}b+\frac{10}{7}x^{7}e^{2}d^{3}a+\frac{1}{4}x^{4}d^{5}b+\frac{5}{4}x^{4}ed^{4}a+xd^{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*e^5*c + 5/19*x^19*e^4*d*c + 1/19*x^19*e^5*b + 5/8*x^16*e^3*d^2*c
+ 5/16*x^16*e^4*d*b + 1/16*x^16*e^5*a + 10/13*x^13*e^2*d^3*c + 10/13*x^13*e
^3*d^2*b + 5/13*x^13*e^4*d*a + 1/2*x^10*e*d^4*c + x^10*e^2*d^3*b + x^10*e^3
*d^2*a + 1/7*x^7*d^5*c + 5/7*x^7*e*d^4*b + 10/7*x^7*e^2*d^3*a + 1/4*x^4*d^5
*b + 5/4*x^4*e*d^4*a + x*d^5*a
```

giac [A] time = 0.34, size = 173, normalized size = 1.06

$$\frac{1}{22}cx^{22}e^{5}+\frac{5}{19}cdx^{19}e^{4}+\frac{1}{19}bx^{19}e^{5}+\frac{5}{8}cd^{2}x^{16}e^{3}+\frac{5}{16}bdx^{16}e^{4}+\frac{1}{16}ax^{16}e^{5}+\frac{10}{13}cd^{3}x^{13}e^{2}+\frac{10}{13}bd^{2}x^{13}e^{3}+\frac{5}{13}adx^{13}e^{4}+\frac{1}{2}cd^{4}x^{10}e+bd^{3}x^{10}e^{2}+ad^{2}x^{10}e^{3}+\frac{1}{7}cd^{5}x^{7}+\frac{5}{7}bd^{4}x^{7}e+\frac{10}{7}ad^{3}x^{7}e^{2}+\frac{1}{4}bd^{5}x^{4}+\frac{5}{4}ad^{4}x^{4}e+ad^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/22*c*x^22*e^5 + 5/19*c*d*x^19*e^4 + 1/19*b*x^19*e^5 + 5/8*c*d^2*x^16*e^3
+ 5/16*b*d*x^16*e^4 + 1/16*a*x^16*e^5 + 10/13*c*d^3*x^13*e^2 + 10/13*b*d^2*
x^13*e^3 + 5/13*a*d*x^13*e^4 + 1/2*c*d^4*x^10*e + b*d^3*x^10*e^2 + a*d^2*x^
10*e^3 + 1/7*c*d^5*x^7 + 5/7*b*d^4*x^7*e + 10/7*a*d^3*x^7*e^2 + 1/4*b*d^5*x
^4 + 5/4*a*d^4*x^4*e + a*d^5*x
```

maple [A] time = 0.00, size = 169, normalized size = 1.04

$$\frac{ce^{5}x^{22}}{22}+\frac{\left(e^{5}b+5de^{4}c\right)x^{19}}{19}+\frac{\left(e^{5}a+5de^{4}b+10d^{2}e^{3}c\right)x^{16}}{16}+\frac{\left(5de^{4}a+10d^{2}e^{3}b+10d^{3}e^{2}c\right)x^{13}}{13}+\frac{\left(10d^{2}e^{3}a+10d^{3}e^{2}b+5d^{4}ec\right)x^{10}}{10}+ad^{5}x+\frac{\left(10ad^{3}e^{2}+5d^{4}eb+cd^{5}\right)x^{7}}{7}+\frac{\left(5d^{4}ea+d^{5}b\right)x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/22*c*e^5*x^22+1/19*(b*e^5+5*c*d*e^4)*x^19+1/16*(a*e^5+5*b*d*e^4+10*c*d^2*
e^3)*x^16+1/13*(5*a*d*e^4+10*b*d^2*e^3+10*c*d^3*e^2)*x^13+1/10*(10*a*d^2*e^
3+10*b*d^3*e^2+5*c*d^4*e)*x^10+1/7*(10*a*d^3*e^2+5*b*d^4*e+c*d^5)*x^7+1/4*(
5*a*d^4*e+b*d^5)*x^4+a*d^5*x
```

maxima [A] time = 0.81, size = 166, normalized size = 1.02

$$\frac{1}{22}ce^{5}x^{22}+\frac{1}{19}\left(5cde^{4}+be^{5}\right)x^{19}+\frac{1}{16}\left(10cd^{2}e^{3}+5bde^{4}+ae^{5}\right)x^{16}+\frac{5}{13}\left(2cd^{3}e^{2}+2bd^{2}e^{3}+ade^{4}\right)x^{13}+\frac{1}{2}\left(cd^{4}e+2bd^{3}e^{2}+2ad^{2}e^{3}\right)x^{10}+\frac{1}{7}\left(cd^{5}+5bd^{4}e+10ad^{3}e^{2}\right)x^{7}+ad^{5}x+\frac{1}{4}\left(bd^{5}+5ad^{4}e\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

mupad [B] time = 1.60, size = 158, normalized size = 0.97

$$x^{4}\left(\frac{bd^{5}}{4}+\frac{5aed^{4}}{4}\right)+x^{19}\left(\frac{be^{5}}{19}+\frac{5cde^{4}}{19}\right)+x^{7}\left(\frac{cd^{5}}{7}+\frac{5bd^{4}e}{7}+\frac{10ad^{3}e^{2}}{7}\right)+x^{16}\left(\frac{5cd^{2}e^{3}}{8}+\frac{5bde^{4}}{16}+\frac{ae^{5}}{16}\right)+\frac{ce^{5}x^{22}}{22}+ad^{5}x+\frac{d^{2}ex^{10}\left(cd^{2}+2bde+2ae^{2}\right)}{2}+\frac{5de^{2}x^{13}\left(2cd^{2}+2bde+ae^{2}\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^19*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7*
((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^16*((a*e^5)/16 + (5*c*d^
```

```
2*e^3)/8 + (5*b*d*e^4)/16) + (c*e^5*x^22)/22 + a*d^5*x + (d^2*e*x^10*(2*a*e
^2 + c*d^2 + 2*b*d*e))/2 + (5*d*e^2*x^13*(a*e^2 + 2*c*d^2 + 2*b*d*e))/13
```

sympy [A] time = 0.10, size = 187, normalized size = 1.15

$$ad^5x + \frac{ce^5x^{22}}{22} + x^{19}\left(\frac{be^5}{19} + \frac{5cde^4}{19}\right) + x^{16}\left(\frac{ae^5}{16} + \frac{5bde^4}{16} + \frac{5cd^2e^3}{8}\right) + x^{13}\left(\frac{5ade^4}{13} + \frac{10bd^2e^3}{13} + \frac{10cd^3e^2}{13}\right) + x^{10}\left(ad^2e^3 + bd^3e^2 + \frac{cd^4e}{2}\right) + x^7\left(\frac{10ad^3e^2}{7} + \frac{5bd^4e}{7} + \frac{cd^5}{7}\right) + x^4\left(\frac{5ad^4e}{4} + \frac{bd^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e
**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**
2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*
e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e
/4 + b*d**5/4)
```

3.2 $\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=135

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+$$

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx = \int \left(ad^4+d^3(bd+4ae)x^3+d^2\left(cd^2+4bde+6ae^2\right)x^6+2de\left(2cd^2+e(3bd+2\right.\right.$$
$$= ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+e(3bd+\right.$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.00

$$\frac{1}{13}e^2x^{13}\left(ae^2+4bde+6cd^2\right)+\frac{1}{5}dex^{10}\left(2ae^2+3bde+2cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e +
 a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)^4*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.64, size = 147, normalized size = 1.09

$$\frac{1}{19}x^{19}e^4c+\frac{1}{4}x^{16}e^3dc+\frac{1}{16}x^{16}e^4b+\frac{6}{13}x^{13}e^2d^2c+\frac{4}{13}x^{13}e^3db+\frac{1}{13}x^{13}e^4a+\frac{2}{5}x^{10}ed^3c+\frac{3}{5}x^{10}e^2d^2b+\frac{2}{5}x^{10}e^3da+\frac{1}{7}x^7d^4c+\frac{4}{7}x^7ed^3b+\frac{6}{7}x^7e^2d^2a+\frac{1}{4}x^4d^4b+x^4ed^3a+xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*e^4*c + 1/4*x^16*e^3*d*c + 1/16*x^16*e^4*b + 6/13*x^13*e^2*d^2*c
+ 4/13*x^13*e^3*d*b + 1/13*x^13*e^4*a + 2/5*x^10*e*d^3*c + 3/5*x^10*e^2*d^2
*b + 2/5*x^10*e^3*d*a + 1/7*x^7*d^4*c + 4/7*x^7*e*d^3*b + 6/7*x^7*e^2*d^2*a
 + 1/4*x^4*d^4*b + x^4*e*d^3*a + x*d^4*a
```

giac [A] time = 0.34, size = 141, normalized size = 1.04

$$\frac{1}{19}cx^{19}e^4+\frac{1}{4}cdx^{16}e^3+\frac{1}{16}bx^{16}e^4+\frac{6}{13}cd^2x^{13}e^2+\frac{4}{13}bdx^{13}e^3+\frac{1}{13}ax^{13}e^4+\frac{2}{5}cd^3x^{10}e+\frac{3}{5}bd^2x^{10}e^2+\frac{2}{5}adx^{10}e^3+\frac{1}{7}cd^4x^7+\frac{4}{7}bd^3x^7e+\frac{6}{7}ad^2x^7e^2+\frac{1}{4}bd^4x^4+ad^3x^4e+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2*x^13*e^2
+ 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*e + 3/5*b*d^2*x^10*e
^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b*d^3*x^7*e + 6/7*a*d^2*x^7*e^2
 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*d^4*x
```

maple [A] time = 0.00, size = 136, normalized size = 1.01

$$\frac{c\,e^4x^{19}}{19}+\frac{\left(e^4b+4d\,e^3c\right)x^{16}}{16}+\frac{\left(e^4a+4d\,e^3b+6e^2d^2c\right)x^{13}}{13}+\frac{\left(4d\,e^3a+6e^2d^2b+4c\,d^3e\right)x^{10}}{10}+\frac{\left(6a\,d^2e^2+4b\,d^3e+c\,d^4\right)x^7}{7}+a\,d^4x+\frac{\left(4d^3ea+d^4b\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3+6*c*d^2*e
^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/7*(6*a*d^2*e^2+4*b*d
^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d^4*x
```

maxima [A] time = 0.72, size = 135, normalized size = 1.00

$$\frac{1}{19}ce^4x^{19}+\frac{1}{16}\left(4\,cde^3+be^4\right)x^{16}+\frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13}+\frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10}+\frac{1}{7}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^7+ad^4x+\frac{1}{4}\left(bd^4+4\,ad^3e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

mupad [B] time = 0.06, size = 130, normalized size = 0.96

$$x^4\left(\frac{b\,d^4}{4}+a\,e\,d^3\right)+x^{16}\left(\frac{b\,e^4}{16}+\frac{c\,d\,e^3}{4}\right)+x^7\left(\frac{c\,d^4}{7}+\frac{4\,b\,d^3\,e}{7}+\frac{6\,a\,d^2\,e^2}{7}\right)+x^{13}\left(\frac{6\,c\,d^2\,e^2}{13}+\frac{4\,b\,d\,e^3}{13}+\frac{a\,e^4}{13}\right)+\frac{c\,e^4\,x^{19}}{19}+a\,d^4\,x+\frac{d\,e\,x^{10}\,\left(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)/
7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/13
+ (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c*d^
2 + 3*b*d*e))/5
```

sympy [A] time = 0.09, size = 151, normalized size = 1.12

$$ad^4x+\frac{ce^4x^{19}}{19}+x^{16}\left(\frac{be^4}{16}+\frac{cde^3}{4}\right)+x^{13}\left(\frac{ae^4}{13}+\frac{4bde^3}{13}+\frac{6cd^2e^2}{13}\right)+x^{10}\left(\frac{2ade^3}{5}+\frac{3bd^2e^2}{5}+\frac{2cd^3e}{5}\right)+x^7\left(\frac{6ad^2e^2}{7}+\frac{4bd^3e}{7}+\frac{cd^4}{7}\right)+x^4\left(ad^3e+\frac{bd^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4
/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e*
*2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x
**4*(a*d**3*e + b*d**4/4)
```

3.3 $\int \left(d+e x^3\right)^3 \left(a+b x^3+c x^6\right) \, dx$

Optimal. Leaf size=103

$$\frac{1}{10} e x^{10} \left(e(ae+3bd)+3cd^2\right)+\frac{1}{7} d x^7 \left(3e(ae+bd)+cd^2\right)+\frac{1}{4} d^2 x^4 (3ae+bd)+a d^3 x+\frac{1}{13} e^2 x^{13} (be+3cd)+\frac{1}{16} c e^3 x^{16}$$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{10} e x^{10} \left(e(ae+3bd)+3cd^2\right)+\frac{1}{7} d x^7 \left(3e(ae+bd)+cd^2\right)+\frac{1}{4} d^2 x^4 (3ae+bd)+a d^3 x+\frac{1}{13} e^2 x^{13} (be+3cd)+\frac{1}{16} c e^3 x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+e x^3\right)^3 \left(a+b x^3+c x^6\right) \, dx &= \int \left(a d^3+d^2 (bd+3ae) x^3+d \left(cd^2+3e(bd+ae)\right) x^6+e \left(3cd^2+e(3bd+ae\right.\right.\\ &= a d^3 x+\frac{1}{4} d^2 (bd+3ae) x^4+\frac{1}{7} d \left(cd^2+3e(bd+ae)\right) x^7+\frac{1}{10} e \left(3cd^2+e(3bd+\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 1.01

$$\frac{1}{10} e x^{10} \left(ae^2+3bde+3cd^2\right)+\frac{1}{7} d x^7 \left(3ae^2+3bde+cd^2\right)+\frac{1}{4} d^2 x^4 (3ae+bd)+a d^3 x+\frac{1}{13} e^2 x^{13} (be+3cd)+\frac{1}{16} c e^3 x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+e x^3\right)^3 \left(a+b x^3+c x^6\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)^3*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.86, size = 112, normalized size = 1.09

$$\frac{1}{16}x^{16}e^3c+\frac{3}{13}x^{13}e^2dc+\frac{1}{13}x^{13}e^3b+\frac{3}{10}x^{10}ed^2c+\frac{3}{10}x^{10}e^2db+\frac{1}{10}x^{10}e^3a+\frac{1}{7}x^7d^3c+\frac{3}{7}x^7ed^2b+\frac{3}{7}x^7e^2da+\frac{1}{4}x^4d^3b+\frac{3}{4}x^4ed^2a+xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^3*c + 3/13*x^13*e^2*d*c + 1/13*x^13*e^3*b + 3/10*x^10*e*d^2*c +
 3/10*x^10*e^2*d*b + 1/10*x^10*e^3*a + 1/7*x^7*d^3*c + 3/7*x^7*e*d^2*b + 3/
7*x^7*e^2*d*a + 1/4*x^4*d^3*b + 3/4*x^4*e*d^2*a + x*d^3*a
```

giac [A] time = 0.30, size = 109, normalized size = 1.06

$$\frac{1}{16}cx^{16}e^3+\frac{3}{13}cdx^{13}e^2+\frac{1}{13}bx^{13}e^3+\frac{3}{10}cd^2x^{10}e+\frac{3}{10}bdx^{10}e^2+\frac{1}{10}ax^{10}e^3+\frac{1}{7}cd^3x^7+\frac{3}{7}bd^2x^7e+\frac{3}{7}adx^7e^2+\frac{1}{4}bd^3x^4+\frac{3}{4}ad^2x^4e+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^2*x^10*e +
 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 + 3/7*b*d^2*x^7*e + 3/
7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4*e + a*d^3*x
```

maple [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{c\,e^3x^{16}}{16}+\frac{\left(e^3b+3cd\,e^2\right)x^{13}}{13}+\frac{\left(e^3a+3bd\,e^2+3d^2ec\right)x^{10}}{10}+\frac{\left(3ad\,e^2+3b\,d^2e+c\,d^3\right)x^7}{7}+a\,d^3x+\frac{\left(3a\,d^2e+b\,d^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e
)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*
x
```

maxima [A] time = 0.65, size = 102, normalized size = 0.99

$$\frac{1}{16}ce^3x^{16}+\frac{1}{13}\left(3\,cde^2+be^3\right)x^{13}+\frac{1}{10}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^{10}+\frac{1}{7}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^7+ad^3x+\frac{1}{4}\left(bd^3+3\,ad^2e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

mupad [B] time = 0.04, size = 102, normalized size = 0.99

$$x^4\left(\frac{b\,d^3}{4}+\frac{3\,a\,e\,d^2}{4}\right)+x^{13}\left(\frac{b\,e^3}{13}+\frac{3\,c\,d\,e^2}{13}\right)+x^7\left(\frac{c\,d^3}{7}+\frac{3\,b\,d^2\,e}{7}+\frac{3\,a\,d\,e^2}{7}\right)+x^{10}\left(\frac{3\,c\,d^2\,e}{10}+\frac{3\,b\,d\,e^2}{10}+\frac{a\,e^3}{10}\right)+\frac{c\,e^3\,x^{16}}{16}+a\,d^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7*
((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e^2
)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x
```

sympy [A] time = 0.09, size = 117, normalized size = 1.14

$$ad^3x + \frac{ce^3x^{16}}{16} + x^{13}\left(\frac{be^3}{13} + \frac{3cde^2}{13}\right) + x^{10}\left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10}\right) + x^7\left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7}\right) + x^4\left(\frac{3ad^2e}{4} + \frac{bd^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e
**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7
 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

3.4 $\int \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=73

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)dx &= \int \left(ad^2+d(bd+2ae)x^3+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^9+ce^2x^{12}\right)dx\\ &= ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)^2*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.91, size = 76, normalized size = 1.04

$$\frac{1}{13}x^{13}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{4}x^4d^2b+\frac{1}{2}x^4eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/7*x^7*d^2*c + 2/7*x^
7*e*d*b + 1/7*x^7*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + x*d^2*a
```

giac [A] time = 0.33, size = 76, normalized size = 1.04

$$\frac{1}{13}\,cx^{13}e^{2}+\frac{1}{5}\,cdx^{10}e+\frac{1}{10}\,bx^{10}e^{2}+\frac{1}{7}\,cd^{2}x^{7}+\frac{2}{7}\,bdx^{7}e+\frac{1}{7}\,ax^{7}e^{2}+\frac{1}{4}\,bd^{2}x^{4}+\frac{1}{2}\,adx^{4}e+ad^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^7 + 2/7*b*
d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + a*d^2*x
```

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{c\,e^{2}x^{13}}{13}+\frac{\left(b\,e^{2}+2cde\right)x^{10}}{10}+\frac{\left(a\,e^{2}+2deb+c\,d^{2}\right)x^{7}}{7}+a\,d^{2}x+\frac{\left(2dea+b\,d^{2}\right)x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4
*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

maxima [A] time = 0.75, size = 69, normalized size = 0.95

$$\frac{1}{13}\,ce^{2}x^{13}+\frac{1}{10}\left(2\,cde+be^{2}\right)x^{10}+\frac{1}{7}\left(cd^{2}+2\,bde+ae^{2}\right)x^{7}+\frac{1}{4}\left(bd^{2}+2\,ade\right)x^{4}+ad^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

mupad [B] time = 0.04, size = 70, normalized size = 0.96

$$x^{7}\left(\frac{c\,d^{2}}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^{2}}{7}\right)+x^{4}\left(\frac{b\,d^{2}}{4}+\frac{a\,e\,d}{2}\right)+x^{10}\left(\frac{b\,e^{2}}{10}+\frac{c\,d\,e}{5}\right)+\frac{c\,e^{2}\,x^{13}}{13}+a\,d^{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^2*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^4*((b*d^2)/4 + (a*d*e)/2) + x
^10*((b*e^2)/10 + (c*d*e)/5) + (c*e^2*x^13)/13 + a*d^2*x
```

sympy [A] time = 0.08, size = 75, normalized size = 1.03

$$ad^{2}x+\frac{ce^{2}x^{13}}{13}+x^{10}\left(\frac{be^{2}}{10}+\frac{cde}{5}\right)+x^{7}\left(\frac{ae^{2}}{7}+\frac{2bde}{7}+\frac{cd^{2}}{7}\right)+x^{4}\left(\frac{ade}{2}+\frac{bd^{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 +
 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

3.5 $\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=42

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1407}

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx&=\int \left(ad+(bd+ae)x^3+(cd+be)x^6+cex^9\right)\,dx\\&=adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
[Out] IntegrateAlgebraic[(d + e*x^3)*(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.03, size = 40, normalized size = 0.95

$$\frac{1}{10}x^{10}ec+\frac{1}{7}x^7dc+\frac{1}{7}x^7eb+\frac{1}{4}x^4db+\frac{1}{4}x^4ea+xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c + 1/7*x^7*d*c + 1/7*x^7*e*b + 1/4*x^4*d*b + 1/4*x^4*e*a + x*d
*a
```

giac [A] time = 0.33, size = 43, normalized size = 1.02

$$\frac{1}{10}\,cx^{10}e+\frac{1}{7}\,cdx^{7}+\frac{1}{7}\,bx^{7}e+\frac{1}{4}\,bdx^{4}+\frac{1}{4}\,ax^{4}e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x^4*e + a*d
*x
```

maple [A] time = 0.00, size = 37, normalized size = 0.88

$$\frac{ce\,x^{10}}{10}+\frac{\left(be+cd\right)x^{7}}{7}+\frac{\left(ae+bd\right)x^{4}}{4}+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

maxima [A] time = 0.55, size = 36, normalized size = 0.86

$$\frac{1}{10}\,cex^{10}+\frac{1}{7}\,(cd+be)x^{7}+\frac{1}{4}\,(bd+ae)x^{4}+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

mupad [B] time = 0.04, size = 38, normalized size = 0.90

$$\frac{c\,e\,x^{10}}{10}+\left(\frac{b\,e}{7}+\frac{c\,d}{7}\right)\,x^7+\left(\frac{a\,e}{4}+\frac{b\,d}{4}\right)\,x^4+a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10
```

sympy [A] time = 0.07, size = 39, normalized size = 0.93

$$adx+\frac{cex^{10}}{10}+x^{7}\left(\frac{be}{7}+\frac{cd}{7}\right)+x^{4}\left(\frac{ae}{4}+\frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-b\right.}{\sqrt{3}d^{2/3}e^{7/3}}$$

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1411, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{x(cd-be)}{e^2}+\frac{cx^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \frac{cx^4}{4e}+\frac{\int \frac{4ae-(4cd-4be)x^3}{d+ex^3}\,dx}{4e}$$

$$= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\left(-a-\frac{d(cd-be)}{e^2}\right)\int \frac{1}{d+ex^3}\,dx$$

$$= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{3d^{2/3}}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}\,x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}\,dx}{3d^{2/3}}$$

$$= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{7/3}}-\frac{\left(cd^2-bde+ae^2\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}\cdots}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\cdots}}{6d^{2/3}e^{7/3}}$$

$$= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{7/3}}-\frac{\left(cd^2-bde+ae^2\right)\log\left(d^{2/3}-\cdots\right.}{6d^{2/3}e^{7/3}}$$

$$= -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\frac{\left(cd^2-bde+ae^2\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{7/3}}+\frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\cdots\right.}{3d^{2/3}e^{7/3}}$$

Mathematica [A] time = 0.15, size = 176, normalized size = 0.94

$$\frac{-\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}+\frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}}+12\sqrt[3]{e}\,x(be-cd)+3ce^{4/3}x^4}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(b
*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*d^
2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(
b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e
^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3), x]
```

fricas [A] time = 0.98, size = 465, normalized size = 2.47

$$\left[\frac{3\,cd^2e^2x^4+6\,\sqrt{\frac{1}{3}}\left(cd^3e-bd^2e^2+ade^3\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3-3\left(d^2e\right)^{\frac{1}{3}}dx-d^2+3\,\sqrt{\frac{1}{3}}\left(2\,dex^2+\left(d^2e\right)^{\frac{2}{3}}x-\left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{ex^3+d}\right)-2\left(cd^2-bde+ae^2\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex^2-\left(d^2e\right)^{\frac{2}{3}}x+\left(d^2e\right)^{\frac{1}{3}}d\right)+4\left(cd^2-bde+ae^2\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex+\left(d^2e\right)^{\frac{2}{3}}\right)-12\left(cd^3e-bd^2e^2\right)x}{12\,d^2e^3},\ \frac{3\,cd^2e^2x^4+12\,\sqrt{\frac{1}{3}}\left(cd^3e-bd^2e^2+ade^3\right)\sqrt{\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(d^2e\right)^{\frac{2}{3}}x-\left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{d^2}\right)-2\left(cd^2-bde+ae^2\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex^2-\left(d^2e\right)^{\frac{2}{3}}x+\left(d^2e\right)^{\frac{1}{3}}d\right)+4\left(cd^2-bde+ae^2\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex+\left(d^2e\right)^{\frac{2}{3}}\right)-12\left(cd^3e-bd^2e^2\right)x}{12\,d^2e^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(-
(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(
2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x
^3 + d)) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2
/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*
x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3), 1/12*(3*c*d^2*e
^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(1/3)/e)
*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/
e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/
3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x
 + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3)]
```

giac [A] time = 0.37, size = 173, normalized size = 0.92

$$-\frac{\sqrt{3}\left(cd^2-bde+ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(cd^2-bde+ae^2\right)e^{(-1)}\log\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}}-\frac{\left(cd^2e^2-bde^3+ae^4\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-4)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3\,d}+\frac{1}{4}\left(cx^4e^3-4\,cdxe^2+4\,bxe^3\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^
(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c*d^2 - b*d*e + a*e^
2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3)
 - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e^(-1))^(1/3)*e^(-4)*log(abs(x - (
-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4)
```

maple [B] time = 0.01, size = 313, normalized size = 1.66

$$\frac{cx^4}{4e}+\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}+\frac{a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{a\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e}-\frac{\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}-\frac{bd\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}+\frac{bd\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}+\frac{bx}{e}+\frac{\sqrt{3}\,cd^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}+\frac{cd^2\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}-\frac{cd^2\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}-\frac{cdx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d),x)
```

```
[Out] 1/4*c*x^4/e+1/e*b*x-1/e^2*c*d*x+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a-1/3/e
^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*d*b+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
c*d^2-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a+1/6/e^2/(d/e)^(
2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*d*b-1/6/e^3/(d/e)^(2/3)*ln(x^2-(d/e)
```

```
^(1/3)*x+(d/e)^(2/3))*c*d^2+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))*a-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e
)^(1/3)*x-1))*d*b+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))*c*d^2
```

maxima [A] time = 1.52, size = 169, normalized size = 0.90

$$\frac{cex^4 - 4\,(cd - be)x}{4\,e^2} + \frac{\sqrt{3}\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} - \frac{\left(cd^2 - bde + ae^2\right)\log\left(x^2 - x\left(\frac{d}{e}\right)^{\frac{1}{3}} + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} + \frac{\left(cd^2 - bde + ae^2\right)\log\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\,e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*e*x^4 - 4*(c*d - b*e)*x)/e^2 + 1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*a
rctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(e^3*(d/e)^(2/3)) - 1/6*
(c*d^2 - b*d*e + a*e^2)*log(x^2 - x*(d/e)^(1/3) + (d/e)^(2/3))/(e^3*(d/e)^(
2/3)) + 1/3*(c*d^2 - b*d*e + a*e^2)*log(x + (d/e)^(1/3))/(e^3*(d/e)^(2/3))
```

mupad [B] time = 0.27, size = 165, normalized size = 0.88

$$x\left(\frac{b}{e} - \frac{c\,d}{e^2}\right) + \frac{c\,x^4}{4\,e} + \frac{\ln\left(e^{1/3}\,x + d^{1/3}\right)\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} + \frac{\ln\left(2\,e^{1/3}\,x - d^{1/3} + \sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3} - 2\,e^{1/3}\,x + \sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c*
d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x -
 d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3
)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2
)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

sympy [A] time = 0.90, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e} + x\left(\frac{b}{e} - \frac{cd}{e^2}\right) + \operatorname{RootSum}\left(27t^3d^2e^7 - a^3e^6 + 3a^2bde^5 - 3a^2cd^2e^4 - 3ab^2d^2e^4 + 6abcd^3e^3 - 3ac^2d^4e^2 + b^3d^3e^3 - 3b^2cd^4e^2 + 3bc^2d^5e - c^3d^6, \left(t \mapsto t\log\left(\frac{3tde^2}{ae^2 - bde + cd^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6
+ 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*
e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*
d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2)
 + x)))
```

3.7 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

Optimal. Leaf size=213

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+$$

Rubi [A] time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(4cd^2-e(2ae+bd)\right)}{3\sqrt{3}d^{5/3}e^{7/3}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\int \frac{cd^2-e(bd+2ae)-3cdex^3}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}e^2} - \frac{\left(4cd^2-e(bd+2ae)\right)\int}{9d^{5/3}}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae\right.}{18}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae\right.}{}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} + \frac{\left(4cd^2-e(bd+2ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}} - \frac{\left(4cd^2-e(bd+2ae\right.}{9d^5}$$

Mathematica [A] time = 0.19, size = 199, normalized size = 0.93

$$\frac{\frac{6\sqrt[3]{e}x\left(e(ae-bd)+cd^2\right)}{d\left(d+ex^3\right)} + \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} - \frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + 18c\sqrt[3]{e}x}{18e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
+ (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))
```

```
/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^3 + cx^6}{\left(d + ex^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3)^2, x]
```

fricas [A] time = 1.08, size = 697, normalized size = 3.27

$$\left[\frac{18\,cd^3e^2x^4 - 3\sqrt{\frac{1}{3}}\left(4\,cd^4e - bd^3e^2 - 2\,ad^2e^3 + \left(4\,cd^3e^2 - bd^2e^3 - 2\,ade^4\right)x^3\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3 - 3\left(d^2e\right)^{\frac{1}{3}}dx - d^2 + 3\sqrt{\frac{1}{3}}\left(2\,dex^2 + \left(d^2e\right)^{\frac{2}{3}}x - \left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{ex^3 + d}\right) + \left(4\,cd^3 - bd^2e - 2\,ade^2 + \left(4\,cd^2e - bde^2 - 2\,ae^3\right)x^3\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex^2 - \left(d^2e\right)^{\frac{2}{3}}x + \left(d^2e\right)^{\frac{1}{3}}d\right) - 2\left(4\,cd^3 - bd^2e - 2\,ade^2 + \left(4\,cd^2e - bde^2 - 2\,ae^3\right)x^3\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex + \left(d^2e\right)^{\frac{2}{3}}\right) + 6\left(4\,cd^4e - bd^3e^2 + ad^2e^3\right)x}{18\left(d^3e^4x^3 + d^4e^3\right)},\ \frac{18\,cd^3e^2x^4 - 6\sqrt{\frac{1}{3}}\left(4\,cd^4e - bd^3e^2 - 2\,ad^2e^3 + \left(4\,cd^3e^2 - bd^2e^3 - 2\,ade^4\right)x^3\right)\sqrt{\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(d^2e\right)^{\frac{2}{3}}x - \left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{d^2}\right) + \left(4\,cd^3 - bd^2e - 2\,ade^2 + \left(4\,cd^2e - bde^2 - 2\,ae^3\right)x^3\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex^2 - \left(d^2e\right)^{\frac{2}{3}}x + \left(d^2e\right)^{\frac{1}{3}}d\right) - 2\left(4\,cd^3 - bd^2e - 2\,ade^2 + \left(4\,cd^2e - bde^2 - 2\,ae^3\right)x^3\right)\left(d^2e\right)^{\frac{2}{3}}\log\left(dex + \left(d^2e\right)^{\frac{2}{3}}\right) + 6\left(4\,cd^4e - bd^3e^2 + ad^2e^3\right)x}{18\left(d^3e^4x^3 + d^4e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3
+ (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*
d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3
)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + (4*c*d^3 - b*
d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(
d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d
*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*
e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e^4*x^3 + d^4*e^3
), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^
3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan
(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2)
 + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d
^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 -
 b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*l
og(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e
^4*x^3 + d^4*e^3)]
```

giac [A] time = 0.38, size = 199, normalized size = 0.93

$$cxe^{(-2)} + \frac{\sqrt{3}\left(4\,cd^2 - bde - 2\,ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{9\left(-de^2\right)^{\frac{2}{3}}d} + \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{18\left(-de^2\right)^{\frac{2}{3}}d} + \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{9\,d^2} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{3\left(x^3e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) + 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*
x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d) + 1/18*
(4*c*d^2 - b*d*e - 2*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-
1))^(2/3))/((-d*e^2)^(2/3)*d) + 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d*e^(-1))
^(1/3)*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^2 + 1/3*(c*d^2*x - b*d*x*e
+ a*x*e^2)*e^(-2)/((x^3*e + d)*d)
```

maple [A] time = 0.01, size = 345, normalized size = 1.62

$$\frac{ax}{3\left(ex^3 + d\right)d} - \frac{bx}{3\left(ex^3 + d\right)e} + \frac{cdx}{3\left(ex^3 + d\right)e^2} + \frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de} + \frac{2a\ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de} - \frac{a\ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de} + \frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2} + \frac{b\ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2} - \frac{b\ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{18\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2} - \frac{4\sqrt{3}\,cd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3} - \frac{4cd\ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3} + \frac{2cd\ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x)
```

```
[Out] c*x/e^2+1/3/d*x/(e*x^3+d)*a-1/3/e*x/(e*x^3+d)*b+1/3/e^2*d*x/(e*x^3+d)*c+2/9
/e/d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/9/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
b-4/9/e^3*d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-1/9/e/d/(d/e)^(2/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*a-1/18/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2
/3))*b+2/9/e^3*d/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c+2/9/e/d/(d
/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a+1/9/e^2/(d/e)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b-4/9/e^3*d/(d/e)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c
```

maxima [A] time = 1.60, size = 204, normalized size = 0.96

$$\frac{\left(cd^2 - bde + ae^2\right)x}{3\left(de^3x^3 + d^2e^2\right)} + \frac{cx}{e^2} - \frac{\sqrt{3}\left(4\,cd^2 - bde - 2\,ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} + \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)\log\left(x^2 - x\left(\frac{d}{e}\right)^{\frac{1}{3}} + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{18\,de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} - \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)\log\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\,de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^3 + d^2*e^2) + c*x/e^2 - 1/9*sqrt(3)
*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(
1/3))/(d*e^3*(d/e)^(2/3)) + 1/18*(4*c*d^2 - b*d*e - 2*a*e^2)*log(x^2 - x*(d
/e)^(1/3) + (d/e)^(2/3))/(d*e^3*(d/e)^(2/3)) - 1/9*(4*c*d^2 - b*d*e - 2*a*e
^2)*log(x + (d/e)^(1/3))/(d*e^3*(d/e)^(2/3))
```

mupad [B] time = 1.80, size = 187, normalized size = 0.88

$$\frac{c\,x}{e^2} + \frac{\ln\left(e^{1/3}\,x + d^{1/3}\right)\left(-4\,c\,d^2 + b\,d\,e + 2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}} + \frac{x\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d\left(e^3\,x^3 + d\,e^2\right)} + \frac{\ln\left(2\,e^{1/3}\,x - d^{1/3} + \sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-4\,c\,d^2 + b\,d\,e + 2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3} - 2\,e^{1/3}\,x + \sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-4\,c\,d^2 + b\,d\,e + 2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)
```

```
[Out] (c*x)/e^2 + (log(e^(1/3)*x + d^(1/3))*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3)) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (log(3^
(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a*e^2 -
 4*c*d^2 + b*d*e))/(9*d^(5/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3
)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3))
```

sympy [A] time = 1.68, size = 206, normalized size = 0.97

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{3d^2e^2 + 3de^3x^3} + \operatorname{RootSum}\left(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^4e^2 - b^3d^3e^3 + 12b^2cd^4e^2 - 48bc^2d^5e + 64c^3d^6, \left(t \mapsto t\log\left(\frac{9td^2e^2}{2ae^2 + bde - 4cd^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Root
Sum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2*e
**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b**3*
d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_
t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

3.8 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1409, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)}{27d^{8/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(e(5ae+bd)+2cd^2\right)}{9\sqrt{3}d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\int \frac{cd^2-e(bd+5ae)-6cdex^3}{\left(d+ex^3\right)^2}\,dx}{6de^2}$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{d+ex^3}\,dx}{9d^2e^2}$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{27d^{8/3}e^2}+\frac{(2}{}$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}}-$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}}-$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)}-\frac{\left(2cd^2+e(bd+5ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}\,d^{8/3}e^{7/3}}-$$

Mathematica [A] time = 0.27, size = 209, normalized size = 0.86

$$\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(5ae+bd)+2cd^2\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)-\frac{3d^{2/3}\sqrt[3]{e}x\left(cd^2\left(4d+7ex^3\right)-e\left(ae\left(8d+5ex^3\right)+bd\left(ex^3-2d\right)\right)\right)}{\left(d+ex^3\right)^2}}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e
*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*A
rcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))*
Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^3 + cx^6}{\left(d + ex^3\right)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3 + c*x^6)/(d + e*x^3)^3, x]
```

fricas [B] time = 0.86, size = 941, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*sqrt(1/3)*(2*c*d^
5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 +
 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log(
(2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(
2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + ((2*c*d^2*
e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3
*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*
x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 +
 b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)
^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)
*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e^2 - b*d^3*
e^3 - 5*a*d^2*e^4)*x^4 - 6*sqrt(1/3)*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 +
 (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5
*a*d^2*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x
- (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + ((2*c*d^2*e^2 + b*d*e^3 + 5
*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 +
5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)
*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^
2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x
+ (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6
+ 2*d^5*e^4*x^3 + d^6*e^3)]
```

giac [A] time = 0.40, size = 224, normalized size = 0.93

$$-\frac{\sqrt{3}\left(2 cd^2 + bde + 5 ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2 x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{27\left(-de^2\right)^{\frac{2}{3}} d^2} - \frac{\left(2 cd^2 + bde + 5 ae^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}} x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{54\left(-de^2\right)^{\frac{2}{3}} d^2} - \frac{\left(2 cd^2 + bde + 5 ae^2\right)\left(-de^{(-1)}\right)^{\frac{1}{3}} e^{(-2)}\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{27 d^3} - \frac{\left(7 cd^2x^4e - bdx^4e^2 - 5 ax^4e^3 + 4 cd^3x + 2 bd^2xe - 8 adxe^2\right)e^{(-2)}}{18\left(x^3e + d\right)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(
-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d^2) - 1/54*(2*c*d^2
+ b*d*e + 5*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3)
)/((-d*e^2)^(2/3)*d^2) - 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d*e^(-1))^(1/3)
*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^3 - 1/18*(7*c*d^2*x^4*e - b*d*x^4
```

```
*e^2 - 5*a*x^4*e^3 + 4*c*d^3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)/((x^3*e
+ d)^2*d^2)
```

maple [A]   time = 0.01, size = 362, normalized size = 1.50

$$\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e}+\frac{5a\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e}-\frac{5a\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d\,e^2}+\frac{b\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d\,e^2}-\frac{b\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\left(\frac{d}{e}\right)^{\frac{2}{3}}d\,e^2}+\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}+\frac{2c\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}-\frac{c\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}e^3}+\frac{\frac{\left(5a\,e^2+deb-7c\,d^2\right)x^4}{18d^2e}+\frac{\left(4a\,e^2-deb-2c\,d^2\right)x}{9d\,e^2}}{\left(e\,x^3+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x)
```

```
[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x
)/(e*x^3+d)^2+5/27/e/d^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/27/e^2/d/(d/e)^(
2/3)*ln(x+(d/e)^(1/3))*b+2/27/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-5/54/e/d^
2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a-1/54/e^2/d/(d/e)^(2/3)*ln
(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b-1/27/e^3/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x
+(d/e)^(2/3))*c+5/27/e/d^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^
(1/3)*x-1))*a+1/27/e^2/d/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))*b+2/27/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*
x-1))*c
```

maxima [A]   time = 1.69, size = 240, normalized size = 0.99

$$-\frac{\left(7\,cd^2e-bde^2-5\,ae^3\right)x^4+2\left(2\,cd^3+bd^2e-4\,ade^2\right)x}{18\left(d^2e^4x^6+2\,d^3e^3x^3+d^4e^2\right)}+\frac{\sqrt{3}\left(2\,cd^2+bde+5\,ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{27\,d^2e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\left(2\,cd^2+bde+5\,ae^2\right)\log\left(x^2-x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\,d^2e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\left(2\,cd^2+bde+5\,ae^2\right)\log\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\,d^2e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*c*d^2*e - b*d*e^2 - 5*a*e^3)*x^4 + 2*(2*c*d^3 + b*d^2*e - 4*a*d*e
^2)*x)/(d^2*e^4*x^6 + 2*d^3*e^3*x^3 + d^4*e^2) + 1/27*sqrt(3)*(2*c*d^2 + b*
d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x - (d/e)^(1/3))/(d/e)^(1/3))/(d^2*e^3
*(d/e)^(2/3)) - 1/54*(2*c*d^2 + b*d*e + 5*a*e^2)*log(x^2 - x*(d/e)^(1/3) +
(d/e)^(2/3))/(d^2*e^3*(d/e)^(2/3)) + 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*log(x
 + (d/e)^(1/3))/(d^2*e^3*(d/e)^(2/3))
```

mupad [B]   time = 0.29, size = 221, normalized size = 0.91

$$\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}-\frac{\frac{x\left(2\,c\,d^2+b\,d\,e-4\,a\,e^2\right)}{9\,d\,e^2}-\frac{x^4\left(-7\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{18\,d^2\,e}}{d^2+2\,d\,e\,x^3+e^2\,x^6}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}-\frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)
```

```
[Out] (log(e^(1/3)*x + d^(1/3))*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
 - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 + b
*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (log(3^(1/2)*d^(1/3)*1i +
2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/
(27*d^(8/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
```

sympy [A]   time = 5.23, size = 246, normalized size = 1.02

$$\frac{x^4\left(5ae^3+bde^2-7cd^2e\right)+x\left(8ade^2-2bd^2e-4cd^3\right)}{18d^4e^2+36d^3e^3x^3+18d^2e^4x^6}+\operatorname{RootSum}\left(19683t^3d^8e^7-125a^3e^6-75a^2bde^5-150a^2cd^2e^4-15ab^2d^2e^4-60abcd^3e^3-60ac^2d^4e^2-b^3d^3e^3-6b^2cd^4e^2-12bc^2d^5e-8c^3d^6,\left(t\mapsto t\log\left(\frac{27td^3e^2}{5ae^2+bde+2cd^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c
*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(19
683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e*
*4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*
d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t,
 _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=132

$$-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+$$

Rubi [A] time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{cd-be}{c^2}+\frac{ex}{c}-\frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)\\
&= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\operatorname{Subst}\left(\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3c^2}\\
&= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^3}+\frac{\left(b^2cd-2ac^2d-b^3e+\right.}{}\\
&= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)}{}\\
&= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{\left(bcd-b^2e+ace\right)\text{lo}}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 126, normalized size = 0.95

$$\frac{\left(-ace+b^2e-bcd\right)\log\left(a+bx^3+cx^6\right)+\frac{2\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^3(cd-be)+c^2ex^6}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*
d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.77, size = 430, normalized size = 3.26

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)ex^6+2\left(\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e\right)x^3+\sqrt{b^2-4\,ac}\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-\left(\left(b^3c-4\,abc^2\right)d-\left(b^4-5\,ab^2c+4\,a^2c^2\right)e\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^3-4\,ac^4\right)},\ \frac{\left(b^2c^2-4\,ac^3\right)ex^6+2\left(\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e\right)x^3-2\sqrt{-b^2+4\,ac}\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(\left(b^3c-4\,abc^2\right)d-\left(b^4-5\,ab^2c+4\,a^2c^2\right)e\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^3-4\,ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*
```

```
log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))
/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*a*c^3
)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*sqrt(-b
^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 1.00, size = 131, normalized size = 0.99

$$\frac{cx^6e + 2\,cdx^3 - 2\,bx^3e}{6\,c^2} - \frac{\left(bcd - b^2e + ace\right)\log\left(cx^6 + bx^3 + a\right)}{6\,c^3} + \frac{\left(b^2cd - 2\,ac^2d - b^3e + 3\,abce\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*log
(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arc
tan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 260, normalized size = 1.97

$$\frac{e\,x^6}{6c} - \frac{be\,x^3}{3c^2} + \frac{d\,x^3}{3c} + \frac{abe\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{2ad\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c} - \frac{b^3e\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c^3} + \frac{b^2d\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c^2} - \frac{ae\ln\left(c\,x^6+b\,x^3+a\right)}{6c^2} + \frac{b^2e\ln\left(c\,x^6+b\,x^3+a\right)}{6c^3} - \frac{bd\ln\left(c\,x^6+b\,x^3+a\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*x^6/c-1/3/c^2*b*e*x^3+1/3/c*d*x^3-1/6/c^2*ln(c*x^6+b*x^3+a)*a*e+1/6/c
^3*ln(c*x^6+b*x^3+a)*b^2*e-1/6/c^2*ln(c*x^6+b*x^3+a)*b*d+1/c^2/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*b*e-2/3/c/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*d-1/3/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.40, size = 3586, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d^3
- b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2*d^2
*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d*e^2
+ 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*b^2*c^
```

```
4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 1
2*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 -
24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*
c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b^3*c^7*d - 45*
b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*
a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3
))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) -
 (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e -
 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4
 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c -
b^2)^(1/2)) + (3*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(4*c^3*(4*a*c - b
^2)*(36*a*c^4 - 9*b^2*c^3))))/(4*a^2*c) - ((2*a*c - b^2)*((((((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1
/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36
*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d -
15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d
^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*
e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3
*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4
 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^
2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*
e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b
*c*e)^3)/(4*c^6*(4*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) - (b*
((a*b^7*e^3 - a*b^4*c^3*d^3 - 4*a^2*b^5*c*e^3 - 2*a^4*b*c^3*e^3 + a^4*c^4*d
*e^2 + a^2*b^2*c^4*d^3 + 5*a^3*b^3*c^2*e^3 - 3*a*b^6*c*d*e^2 + 3*a*b^5*c^2*
d^2*e + 2*a^3*b*c^4*d^2*e - 6*a^2*b^3*c^3*d^2*e + 9*a^2*b^4*c^2*d*e^2 - 7*a
^3*b^2*c^3*d*e^2)/c^6 + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*a*b^5*c
^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30*a*b^4*
c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a
*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3
*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*
b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*
e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*
b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*
b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b^3*e + 2*
a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a*b*(b^3*e
 + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 1
2*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*
(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (3
*a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e + 12*a^2*c^2*e -
3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*c^3*(4*a*c - b^2)*(36*a*c^4 -
9*b^2*c^3))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^8*d - 72*a*b^2*c^7*
d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b
^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a
*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^
3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^
2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)) + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*
```

```
a*b^5*c^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30
*a*b^4*c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d
 + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^
2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c
- b^2)^(1/2)) - (a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^3)/(2*c^6*(4
*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^9*e^3 + 8*a^3*c^6*d
^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 + 3*b^7*c^2*d^2*e - 12*a^2*b^2*c^5*d^3 +
 27*a^2*b^5*c^2*e^3 - 27*a^3*b^3*c^3*e^3 - 9*a*b^7*c*e^3 - 3*b^8*c*d*e^2 -
21*a*b^5*c^3*d^2*e + 24*a*b^6*c^2*d*e^2 - 36*a^3*b*c^5*d^2*e + 48*a^2*b^3*c
^4*d^2*e - 63*a^2*b^4*c^3*d*e^2 + 54*a^3*b^2*c^4*d*e^2))*(b^3*e + 2*a*c^2*d
 - b^2*c*d - 3*a*b*c*e))/(3*c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 55.47, size = 620, normalized size = 4.70

$$x^{3}\left(-\frac{be}{3c^{2}}+\frac{d}{3c}\right)+\left(-\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)\log\left(x^{3}+\frac{2a^{2}ce-ab^{2}e+abcd+12ac^{3}\left(-\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)-3b^{2}c^{2}\left(-\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)}{3abce-2ac^{2}d-b^{3}e+b^{2}cd}\right)+\left(\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)\log\left(x^{3}+\frac{2a^{2}ce-ab^{2}e+abcd+12ac^{3}\left(\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)-3b^{2}c^{2}\left(\frac{\sqrt{-4ac+b^{2}}\left(3abce-2ac^{2}d-b^{3}e+b^{2}cd\right)}{6c^{3}\left(4ac-b^{2}\right)}-\frac{ace-b^{2}e+bcd}{6c^{3}}\right)}{3abce-2ac^{2}d-b^{3}e+b^{2}cd}\right)+\frac{ex^{6}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] x**3*(-b*e/(3*c**2) + d/(3*c)) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**
2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)
/(6*c**3))*log(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqrt(-
4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c -
 b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(-sqrt(-4*a*c +
b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2))
- (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b*
*2*c*d)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d
)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (
2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e
 - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*
e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c*
*2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d
)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) + e*x**6/(6*c)
```

3.10 $\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=97

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 773, 634, 618, 206, 628}

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(d+ex)}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{ex^3}{3c}+\frac{\operatorname{Subst}\left(\int \frac{-ae+(cd-be)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\
&= \frac{ex^3}{3c}+\frac{(cd-be)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}-\frac{\left(bcd-b^2e+2ace\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \right.}{6c^2}\\
&= \frac{ex^3}{3c}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{\left(bcd-b^2e+2ace\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2\right.}{3c^2}\\
&= \frac{ex^3}{3c}+\frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$
\frac{\frac{2\left(-2ace+b^2e-bcd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(cd-be)\log\left(a+bx^3+cx^6\right)+2cex^3}{6c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.16, size = 305, normalized size = 3.14

$$
\left[\frac{2\left(b^2c-4ac^2\right)ex^3+\left(bcd-\left(b^2-2ac\right)e\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)+\left(\left(b^2c-4ac^2\right)d-\left(b^3-4abc\right)e\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^2-4ac^3\right)}, \frac{2\left(b^2c-4ac^2\right)ex^3+2\left(bcd-\left(b^2-2ac\right)e\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(\left(b^2c-4ac^2\right)d-\left(b^3-4abc\right)e\right)\log\left(cx^6+bx^3+a\right)}{6\left(b^2c^2-4ac^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + 2*(
```

```
b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.07, size = 95, normalized size = 0.98

$$\frac{x^3e}{3\,c}+\frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$\frac{e\,x^3}{3c}-\frac{2ae\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c}+\frac{b^2e\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c^2}-\frac{bd\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;c}-\frac{be\ln\left(c\,x^6+b\,x^3+a\right)}{6c^2}+\frac{d\ln\left(c\,x^6+b\,x^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*e*x^3/c-1/6/c^2*ln(c*x^6+b*x^3+a)*b*e+1/6/c*ln(c*x^6+b*x^3+a)*d-2/3/c/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*e+1/3/c^2/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/3/c/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.95, size = 2624, normalized size = 27.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] (e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d -
 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/2)
*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^3*c
^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^2*d*
e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^
3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4
*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^3*c^4*e +
36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*
```

```
c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*b^
2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(4*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((45*b^2*c^5*d
 - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3
*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/
(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e +
 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) + (b^2*(2*a*c*e - b^2*e + b*c*d)^3)/(4*c^3*(4*a*c - b^2)^(3/2)
) - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^3*e^2 -
 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4*e + 36
*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*
e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e)
)/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) + (b*((a^2*b^2*c*e^3 - a*b^4*e^3
 + a^2*c^3*d^2*e + a*b*c^3*d^3 + 3*a*b^3*c*d*e^2 - 3*a*b^2*c^2*d^2*e - 2*a^
2*b*c^2*d*e^2)/c^3 - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^
2 + 12*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d
 - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((36*a^2*c^5*e + 72*a*b*c^5*d -
72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*
b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*
b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*a*b*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^5*e
 + 72*a*b*c^5*d - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d -
 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d)
)/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e
 + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^2 + 12
*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d - 72*
a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c
*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e
))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b
^2)^(1/2)) + (a*b*(2*a*c*e - b^2*e + b*c*d)^3)/(2*c^3*(4*a*c - b^2)^(3/2)))
)/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(8*a^3*c^3*e^3 - b^6*e^3 + b^3*c^3*d^3 -
3*b^4*c^2*d^2*e - 12*a^2*b^2*c^2*e^3 + 6*a*b^4*c*e^3 + 3*b^5*c*d*e^2 + 6*a*
b^2*c^3*d^2*e - 12*a*b^3*c^2*d*e^2 + 12*a^2*b*c^3*d*e^2))*(2*a*c*e - b^2*e
+ b*c*d))/(3*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 17.49, size = 434, normalized size = 4.47

$$\left(- \frac{\sqrt{-4ac + b^2}\left(2ace - b^2e + bcd\right)}{6c^2\left(4ac - b^2\right)} - \frac{be - cd}{6c^2}\right)\log\left(x^3 + \frac{-abe - 12ac^2\left(- \frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)} - \frac{be-cd}{6c^2}\right) + 2acd + 3b^2c\left(- \frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)} - \frac{be-cd}{6c^2}\right)}{2ace - b^2e + bcd}\right) + \left(\frac{\sqrt{-4ac + b^2}\left(2ace - b^2e + bcd\right)}{6c^2\left(4ac - b^2\right)} - \frac{be - cd}{6c^2}\right)\log\left(x^3 + \frac{-abe - 12ac^2\left(\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)} - \frac{be-cd}{6c^2}\right) + 2acd + 3b^2c\left(\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)} - \frac{be-cd}{6c^2}\right)}{2ace - b^2e + bcd}\right) + \frac{ex^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
(b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2)*
(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2))
+ 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c*
```

```
*2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2)*(2*
a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2
*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(
4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x**
3/(3*c)
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=72

$$\frac{e \log \left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e \log \left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c}\\ &= \frac{e\log\left(a+bx^3+cx^6\right)}{6c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c}\\ &= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^3+cx^6\right)}{6c}\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^3+cx^6\right) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 1.28, size = 216, normalized size = 3.00

$$\left[\frac{\left(b^2-4ac\right)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4ac}\,(2cd-be)\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)}{6\left(b^2c-4ac^2\right)}, \frac{\left(b^2-4ac\right)e\log\left(cx^6+bx^3+a\right)-2\sqrt{-b^2+4ac}\,(2cd-be)\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{6\left(b^2c-4ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a
*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^
6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 1.21, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^6+bx^3+a\right)}{6c} + \frac{(2cd-be)\arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be\arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\,c}+\frac{2d\arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}+\frac{e\ln\left(cx^6+bx^3+a\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b
^2)^(1/2))*d-1/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.63, size = 1632, normalized size = 22.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] - (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
(atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)*(
((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 1
2*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 -
12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2*e -
36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*
(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*
a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*
d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e)*(6*c^3*d^
2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e
 - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^
2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - ((((b*e - 2*c*d
)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2
- 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*e - 12*a*c*e)*(b
*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(
6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(
4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*a*c - b^2)*(((3*b^2
*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*
b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^
2*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c -
b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b^2*(b*e - 2*c*d)^3)/(4*(4*a*c -
b^2)^(3/2)) + ((b*e - 2*c*d)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2
```

```
*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/
(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^2 - 18*b*c^2*d*e))/(6*c*(4*a*c - b^2)
^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/(b^3*e^3 - 8*
c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2) + ((2*a*c - b^2)*(4*a*c - b^2)*((
(3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c
^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) +
 (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*a*c^2 - 9*b^2*c)*(4*a*
c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) + ((b*e - 2*c*d)*(((3*b^2*e - 12
*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*
a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 - 12*a*c^2*d*e))
/(6*c*(4*a*c - b^2)^(1/2)) - (a*b*(b*e - 2*c*d)^3)/(2*(4*a*c - b^2)^(3/2)))
)/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2*
c*d))/(3*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 6.55, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c} - \frac{\sqrt{-4ac + b^2}\,(be - 2cd)}{6c\left(4ac - b^2\right)}\right)\log\left(x^3 + \frac{-12ac\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right) + 2ae + 3b^2\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right) - bd}{be - 2cd}\right) + \left(\frac{e}{6c} + \frac{\sqrt{-4ac + b^2}\,(be - 2cd)}{6c\left(4ac - b^2\right)}\right)\log\left(x^3 + \frac{-12ac\left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right) + 2ae + 3b^2\left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right) - bd}{be - 2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3
 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

3.12 $\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)} dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{d\log(x)}{a}-\frac{d\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}+\frac{(-bd+2ae)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(-bd+2ae)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 80, normalized size = 1.03

$$
\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)), x]
```

fricas [A] time = 1.36, size = 240, normalized size = 3.08

$$
\left[-\frac{\left(b^2-4\,ac\right)d\log\left(cx^6+bx^3+a\right)-6\left(b^2-4\,ac\right)d\log(x)+\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\left(ab^2-4\,a^2c\right)},-\frac{\left(b^2-4\,ac\right)d\log\left(cx^6+bx^3+a\right)-6\left(b^2-4\,ac\right)d\log(x)-2\sqrt{-b^2+4\,ac}\,(bd-2\,ae)\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{6\left(ab^2-4\,a^2c\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c -
(2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), -
```

```
1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 1.05, size = 76, normalized size = 0.97

$$-\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arcta
n((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\,a}+\frac{2e\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}+\frac{d\ln(x)}{a}-\frac{d\ln\left(c\,x^6+b\,x^3+a\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/a*d*ln(x)-1/6*d*ln(c*x^6+b*x^3+a)/a+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3
+b)/(4*a*c-b^2)^(1/2))*e-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-
b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.76, size = 4149, normalized size = 53.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*((((((3*b^2*d - 12*a*c*d)*(
((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*
a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*
a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*
a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 -
 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12
*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 -
 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 3
6*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d - 12*a*c*d))
/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*
c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*
```

```
a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4
*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e
))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^2)^(1/2)) - (((
(2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b
^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c
^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378
*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2)))
)/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^
2*c^4)*(2*a*e - b*d)^2)/(72*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(2*a*e
 - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 3
78*a*b^2*c^4)*(2*a*e - b*d)^3)/(432*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e))/(16
*a^4*c^3*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) - ((c^3*e^4 - ((3*b
^2*d - 12*a*c*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2
*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*
a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*
b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)
) + 7*c^4*d*e^2))/(2*(9*a*b^2 - 36*a^2*c)) + ((((2*a*e - b*d)*(((2*a*e - b*
d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a
^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(
1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/
(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2))
 + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d)^2)/(72
*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c
*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d
 - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12
*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^
2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a
*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(10
8*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^
3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4
*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((108*b^4*c^
3 - 378*a*b^2*c^4)*(2*a*e - b*d)^4)/(1296*a^4*(4*a*c - b^2)^2))*(4*b^5*d -
2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e))/(16
*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e))
))/(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) -
 (3*(4*a*c - b^2)^(3/2)*(c^3*d*e^3 + ((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*
(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 1
2*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3
*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e
 - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(2*(9*a*b^2 - 36*a^2*
c)) - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(
27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a
*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d
*e))/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(2*(9*a*b^2 - 3
6*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^
2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6
*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4
*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) + ((2
*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b
^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^
2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(2*a*e
- b*d))/(6*a*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(2*a*e - b*d)^4)/(48*a^3*(4*a*
c - b^2)^2))*(4*b^5*d - 2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2
*d + 4*a^2*b^2*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) + (3*(
4*a*c - b^2)^2*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d
```

```
)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(
9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12
*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9
*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d -
 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*
c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a
*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(((2*a*e - b*d
)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d -
 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c
^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2
)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a
*e - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(6*a*(4*a*c - b^2)^
(1/2)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(27*b
^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))
/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(6*a*(4*a*c - b^2)^
(1/2)) - (b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d)^3)/(16*a^2*(9*a*b^2 -
36*a^2*c)*(4*a*c - b^2)^(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2
*c*d + 2*a^2*b*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)))*(2*a*
e - b*d))/(3*a*(4*a*c - b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.13 $\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Rubi [A] time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e
)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-acd-abe+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\operatorname{Subst}\left(\int \frac{b^2d-acd-abe+c(bd-ae)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3a^2}\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a^2}+\frac{\left(b^2d-2acd-\right.}{}\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2d-2acd-abe\right)\operatorname{Sul}}{}\\
&= -\frac{d}{3ax^3}-\frac{\left(b^2d-2acd-abe\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(\right.}{6a}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 130, normalized size = 1.16

$$\frac{\operatorname{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{-\#1^3ace\log(x-\#1)+\#1^3bcd\log(x-\#1)-abe\log(x-\#1)-acd\log(x-\#1)+b^2d\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}+\frac{\log(x)(ae-bd)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
& , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[
x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)), x]
```

fricas [A] time = 2.60, size = 385, normalized size = 3.44

$$\left[\frac{\left(abe-\left(b^2-2ac\right)d\right)\sqrt{b^2-4ac}\,x^3\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)+\left(\left(b^3-4abc\right)d-\left(ab^2-4a^2c\right)e\right)x^3\log\left(cx^6+bx^3+a\right)-6\left(\left(b^3-4abc\right)d-\left(ab^2-4a^2c\right)e\right)x^3\log(x)-2\left(ab^2-4a^2c\right)d}{6\left(a^2b^2-4a^3c\right)x^3},\frac{2\left(abe-\left(b^2-2ac\right)d\right)\sqrt{-b^2+4ac}\,x^3\arctan\left(-\frac{\left(2cx^3+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(\left(b^3-4abc\right)d-\left(ab^2-4a^2c\right)e\right)x^3\log\left(cx^6+bx^3+a\right)-6\left(\left(b^3-4abc\right)d-\left(ab^2-4a^2c\right)e\right)x^3\log(x)-2\left(ab^2-4a^2c\right)d}{6\left(a^2b^2-4a^3c\right)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*
c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a))
 + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6
```

```
*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-b^2 +
 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*((b^3 -
 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^
2*b^2 - 4*a^3*c)*x^3)]
```

giac [A] time = 1.07, size = 128, normalized size = 1.14

$$\frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{(bd-ae)\log\left(|x|\right)}{a^2}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bdx^3-ax^3e-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e - a*d)/(a^2*x^3)
```

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$-\frac{be\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a}-\frac{2cd\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a}+\frac{b^2d\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}\;a^2}+\frac{e\ln(x)}{a}-\frac{e\ln\left(c\,x^6+b\,x^3+a\right)}{6a}-\frac{bd\ln(x)}{a^2}+\frac{bd\ln\left(c\,x^6+b\,x^3+a\right)}{6a^2}-\frac{d}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*d/x^3+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/6/a*ln(c*x^6+b*x^3+a)*e+1/6/a^2*
ln(c*x^6+b*x^3+a)*b*d-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2
)^(1/2))*b*e-2/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*
c*d+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.57, size = 7282, normalized size = 65.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - (log(((((((a*e - b*d + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))
/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b + 4*b
^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e
 + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e - b*d
 + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) +
 (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))
/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
```

```
1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(((((((b*d
- a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*
b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a
*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*e + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) + (
3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4*d*(3*a*b*e - 3
*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(
4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3
+ (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^3*(a*e - b*d))/a^4
+ (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(
36*a^3*c - 9*a^2*b^2)) - d/(3*a*x^3) - (atan((48*a^8*x^3*(((((((((18*a^3*b
^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*
d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 +
((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12
*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*
a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/
(2*(36*a^3*c - 9*a^2*b^2)) - (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 2
52*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*
b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*
d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2
*c^4)*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*
b*c*d))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (((7*a^2*c^6*d^2*e - 12*a*b*c^6*d^3)/a
^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18
*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^
4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))
/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b
^2)^(1/2)) - ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)
^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(432*a^10*(4*a*c - b^2)
^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b
^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e))/(16*a^4*c^3*(49*a^3*c*e^2 - 12*b^
4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a
^2*b*c*d*e)) - (((((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5
*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^
2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c
*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*
b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12
*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d)
)/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e
 - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(72*
a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((7*a^2*c^6*d^2*e - 12*a*b*c
^6*d^3)/a^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^
4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a
^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*
d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*
```

```
a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (c^7*d^4)/a^4 - ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)^4)/(1296*a^12*(4*a*c - b^2)^
2) + (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((
108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*
(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d
+ 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c
- b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5
*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252
*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))/(6
*a^2*(4*a*c - b^2)^(1/2)))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b
^2*c^2*d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(64*a^4*c^3*(4
*a*c - b^2)^(1/2)*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2
 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)))*(4*a*c - b^2)^2)/(8*a^
3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^
3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a
^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2) + (3*a^4*(4*a*c - b^2)^2*(((((((27
*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(
3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))
*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e
 - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3*b*c^5*d^2 - 27*a^2*
b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^
3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2
*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/
(6*a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)) + (((a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*
c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d
*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 +
(27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c
 - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
- (((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*
a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9
*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3
*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c
*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2
*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d
 - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e)
)/(c^3*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3
*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*
b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12
*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c
^4*d*e^2)) - (3*a^4*(4*a*c - b^2)^(3/2)*((b*c^6*d^4 - a*c^6*d^3*e)/a^4 - ((
(a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^
2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27
```

```
*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e
 + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a
*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((((((27*a
^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*
b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(
a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e -
 b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(4*
a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2
*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2
*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a
^2*b^2)) + ((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a
^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + (9*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3
*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c
^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e
- b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))
/(6*a^2*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^4)/(48*a^
7*(4*a*c - b^2)^2))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b^2*c^2*
d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(4*c^3*(49*a^3*c*e^2
- 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2
 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*
b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12
*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2)))*(a*b*e -
b^2*d + 2*a*c*d))/(3*a^2*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.14 $\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=723

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-\right.}{}$$

Rubi [A] time = 1.81, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1502, 1510, 292, 31, 634, 617, 204, 628}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\int \frac{x\left(2ae-2(cd-be)x^3\right)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}\cdots}}{2c}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt[3]{b\cdots}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}\cdots}}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2\cdots}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\cdots}{3\,2^{2/3}c^{5}\cdots}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\cdots}{3\,2^{2/3}c^{5}\cdots}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{b+\cdots}}}{\cdots}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{3ex^2-2\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x -
 #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.01, size = 70, normalized size = 0.10

$$\frac{e x^2}{2c} - \frac{\left((be - cd)\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^4 + \mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right) ae\right) \ln\left(-\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right) + x\right)}{3c\left(2\,\mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^5 c + \mathrm{RootOf}\left(\_Z^6 c + \_Z^3 b + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+_R*a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_
R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex^2}{2c} - \frac{-\int \frac{(cd-be)x^4 - aex}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)/
c
```

mupad [B] time = 42.01, size = 13112, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a
*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d
^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*
c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3
- 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*
e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3
```

*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*
e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e -
 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*
e - b^2*e + b*c*d))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b
^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*
a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2
*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^
(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*
c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5
*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*
d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 +
 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2
*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2
 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 +
5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c
^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b
^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^
4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2
*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2

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) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b
^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d
))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a
^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*
b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e +
 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + (e*x^2)/(2*c) + log(- (2^(1/3)*((2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*
a*c*e^2 - 2*b*c*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*
c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d
*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b
^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*
b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-
(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3
*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^
2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*
c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3
*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*
a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*
d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(
-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^
3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e
^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5
*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^
3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a
^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/
2 - 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*
a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3
*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e
+ 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
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 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log(- (2^(1/3)*((2^(2/3)*(3^(1/2)
*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c
*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 1
6*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^
4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 +
96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 +
 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d
^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*(4*a*c - b^2)
*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d
*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(-(b^8*e^3 + 16*
a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*
d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*
d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96
*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6
*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c
*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8
*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3
 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 +
5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*
d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2
*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3
*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 -
 48*a^2*b^2*c^7)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^
2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(
1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 -
 b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*
b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6
*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*
e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*
a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^
3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^
```

```
5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c
^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^
2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2
 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b
^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b
^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*
e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b
^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*
c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c
^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^
2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*
b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2
*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c
^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 -
 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e
 + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*
b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*
c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7
)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(4*a*c -
b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(3
^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 -
b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41
*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^
4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^
2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*
a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*
a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*
d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2
*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^
5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c
```

```
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a
^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72
*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4
*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e
 + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^
3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d
^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^
2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3
*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+b\cdots}{\sqrt{b^2-4ac}}\right.}{}$$

Rubi [A] time = 1.46, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1502, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}}{2c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}}{\frac{\left(b-\sqrt{b^2-4ac}\right)}{2^{2/3}}}}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-\right.}}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ac}{\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{2}\,c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ac}{\sqrt{b^2-4ac}}\right.}{3\sqrt[3]{2}\,c}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(1-\right.}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^3 + d)x^3}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 67, normalized size = 0.09

$$\frac{ex}{c} + \frac{\left((-be + cd)\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^3 - ae\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right) + x\right)}{3c\left(2\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^5 c + \mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/c*e*x+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=Root
Of(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c} - \frac{-\int \frac{(cd-be)x^3 - ae}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

mupad [B] time = 30.15, size = 11453, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d
^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e
+ 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^
2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
```

```
b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2
*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*
a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*
e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^
2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6
*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e
^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^
2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*
c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^
2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3
*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
 + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3
*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^
3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3
*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
- 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 +
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3
- b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b
*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32
*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3
```

```
- 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2
*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2
- 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*d*
x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((
b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c
^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(
1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 +
 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
 b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*e^3
 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a*b*
c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 +
2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4
*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d
*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2
*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^
2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2
*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d
^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e
^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*
d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e
^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))
^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3
*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2
*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*
b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3)
)^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a
^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*
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b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^
2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*
c - b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*
e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a
*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2
+ 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^
2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4
*(4*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3
+ 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c
*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*
c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^
3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e
^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*
e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*
c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c
^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*
d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6
)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81*
a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
+ 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 -
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d
*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*
b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a
*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b
^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*
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c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2
*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*
b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2
*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^
2*c^6)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)
*(81*a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*
c - b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^
2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b
^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2
 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
 - b^2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2
*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*
e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^
4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(c^4*(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3
+ a^2*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d
*e^2 + 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b
^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^
6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 2
7*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^
2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 -
b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e
 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3
- 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^
3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a
^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^
5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a
^2*b^2*c^6)))^(1/3) + (e*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\right.}{}$$

Rubi [A] time = 0.73, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}$$

$$=-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\,2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$=-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\,2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$=-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\,2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$=-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right.}{3}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
 + 2*c*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 119.72, size = 13607, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4
*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*
d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c
^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c +
4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(2*(a*
b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a
^3*b*c^4)*e)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^
4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c
 + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*
c^7)) - sqrt(3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^
2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)
*e^4))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b
^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^
3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(
a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(
a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 -
4*a^2*c^3))^(1/3)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c
^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c
 + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^
2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^
6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12
*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*
b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a
^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 1
2*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^
5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*
a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a
^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*
a^4*b*c^2)*e^5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*
c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e
^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4
*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(
a*b^2*c^2 - 4*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b
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^4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4
*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3
*c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a
*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 +
 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 -
 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b
*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*
b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*
a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2
*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4
*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^
2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 +
2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2
*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)
*e^7)) - (1/2)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d
 - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - sqrt(3)*((b^3*c^2 - 4*a*b*c^3
)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e
^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a
*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(
a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(
7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a
^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a
^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) - sqrt(3)*(b*c^3*
d^5 + 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2
)*d^2*e^3 + (a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5
+ 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^
2*e^3 + (a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5)) - 2/3*sqrt(3)*(
1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqr
t((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(
a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 -
 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^
6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^
2 - 4*a^2*c^3))^(1/3)*arctan(-1/3*((1/2)^(5/6)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a
^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*
sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 -
2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^
4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)
*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + sqrt(
3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*
b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^4))*(-(c^2
*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 -
12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^
2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2
*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 -
 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(
1/3)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c
^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^
2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b
*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^
2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^
3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*
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e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^
3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e
^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5
 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3
*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*
e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36
*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^
5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^
2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3
*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a
^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a
^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4
*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*
b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b
^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*
c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 + ((a*b^5*c^3 - 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*
d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c
^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
+ 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
+ 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2
*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a
^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*
b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64
*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c
^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^
4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^
2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)) - (1/2
)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2
 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
+ 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
+ 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
 48*a^4*b^2*c^6 - 64*a^5*c^7)) + sqrt(3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(
a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 -
 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b
^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6
*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^
3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) + sqrt(3)*(b*c^3*d^5 + 10*a*b*
c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (
a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5 + 10*a*b*c^2*
d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (a*b^
3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5)) - 1/6*(1/2)^(1/3)*(-(c^2*d^3
 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a
*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*
c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*
a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)
*log(2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^
5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3
*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*
e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*
c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48
*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*
```

```
(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*
(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (
a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*
a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 -
 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 -
(a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b
^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*
(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*
d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 +
 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3
*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6
*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c
^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^
3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c -
 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d
^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a*b^5*c^3 - 8*a^2*b^
3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e
+ 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8
*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a
*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7
*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^
2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^
4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 -
 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4
)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3
*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c
+ 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c
^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)) - 1/6*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*
d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^
5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*
e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d
*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c
^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log(2*(b
*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5
*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a
*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^
3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*
a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*
c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^
3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^
2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c
^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*
c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c
+ 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 +
32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*(-(c^2*d^
3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*
a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b
*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^
3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12
*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3
) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^3 - 24*a^
2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^4 -
(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 + ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*
b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3
+ 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*
```

```
c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4
*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6
 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^
3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2
 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2
*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c
^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*
b^2*c^2 - 4*a^2*c^3))^(1/3)) + 1/3*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a
*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(
a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(
7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a
^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a
^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log((1/2)^(2/3)*(
(b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4
*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64
*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (
a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 - ((a*b^6*c^3 - 12*a^2*b^4*c^4 + 4
8*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^
2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^
3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^
2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c
^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*
c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e
^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4
*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(
a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e
+ (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*
(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5
 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x) + 1/3*(1/2)^(1
/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*
c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c
^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2
*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2
*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a
^2*c^3))^(1/3)*log((1/2)^(2/3)*((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3
 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2
- (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3
*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 +
((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*
c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6
- 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*
a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c -
 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4
 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2
 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e +
 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 +
 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5
+ (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 +
48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^
7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b
^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 +
6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 -
 2*a^4*c)*e^7)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^3+d\right)x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.00, size = 49, normalized size = 0.08

$$\frac{\left(\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^4 e+\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^5 c+3\,\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^3+d\right)x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 24.56, size = 7457, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 +
 a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 -
b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(
4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(
2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c -
 b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^
2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a
^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 2
4*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c
 - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*
c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c
^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3
 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*
d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4
```

```
*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3
+ b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (
2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4
*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3
*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2
*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e
^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a
^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27
*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^
2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) - log(c*x*(
b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^
3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a
*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^
2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*
(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(
4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c
^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c
^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2
 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2
*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 10
8*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 + 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3
 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^
2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48
*a^3*b^2*c^4)))^(1/3) - log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^
(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2
*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*
c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
)/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(
```

```
2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*
d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a
*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^
2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c
^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))
^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a
*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*
a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^
3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 +
1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*
c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a
^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5
- a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log(c*x*(b*e - c*d
)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c
^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(1/2)*1i - 1)*(27*c^3*x*(4*a*c - b
^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^
(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 -
8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16
*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 +
b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*
b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*
e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c
^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^
2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*
e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*
c^4)))^(1/3) + log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^
(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*
(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(
1/2)*1i - 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a
*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3
+ 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b
^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2
```

```
+ 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3
*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*
b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2
)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e -
 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*
b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2
 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^
2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c
^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\right.}{6}$$

Rubi [A] time = 0.65, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4\cdots}}$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}\,x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}\,x}{\sqrt[3]{2}}+c^{2/3}x^2}\,d\cdots}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}\cdots$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\cdots\right.}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)\cdots}\cdots$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\cdots\right.}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)\cdots}\cdots$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e+\frac{2cd-\cdots}{\sqrt{b^2-\cdots}}\right.\cdots}{\cdots}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^
2 + 2*c*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{a+bx^3+cx^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(a + b*x^3 + c*x^6), x]
```

fricas [B] time = 39.39, size = 14094, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a
^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a
^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a
^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c
 - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d
*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2
 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*
c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a
*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d
^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2
*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (
a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2
*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*
c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
 + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*
a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*
c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a
^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^
4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*
b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^
3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5
*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3
+ 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a
*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*
c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 -
 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*
c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/
(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7
 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
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+ 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 +
 (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^
(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c
 - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 3
2*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a
^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((
a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^
6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*
d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^
2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3
*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e
 + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6
- (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5
*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d
^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3
)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b
^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d
*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*
d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^
2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3
*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^
3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c
 - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^
3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*
e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(
12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 +
 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b
*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^
2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*
b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 2*sqrt(3)*(a^4*b*e^7 - (b^2*
c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2
*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*
a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d
*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e -
 (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*
e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5
- 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) + 2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c
*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan
(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c
^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^
6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^
3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)) + sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2
 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*
b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a
^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 -
 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 -
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a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^
2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*
a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)
- (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*
e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) +
 sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2
*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^
3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c
- 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*
e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6
 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^
5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*
d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^
4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c +
6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3
*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b
^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c
^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*
(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3
*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 -
 2*(a^4*b^3 - 4*a^5*b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^
3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64
*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(
-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*
b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a
^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2
- 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10
*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d
^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3
- (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2
- 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a
^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*
(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3
- 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2
*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*
c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6
)) + 2*sqrt(3)*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)
*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c
^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*
```

```
d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^
7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
+ (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 1/6*(1/2
)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(
12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 +
 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2
- a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c +
 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17
*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c
- 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c
^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3
*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4
*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12
*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4
*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*
b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^
3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^
4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3)*(((a^2*b^5*c
^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^
5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c
^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 +
 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*
d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e
^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 -
 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c
+ 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 -
 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(
1/3)) - 1/6*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*
a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4
*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^
4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5
)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d
^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^
2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2
)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5
*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 +
 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*
(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 +
((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*
b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*
c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
```

```
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*
c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2
*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^
6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d
^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)
*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1
/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3
*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)
*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*
c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*
c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + ((b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4
*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*
e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*
d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(
-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 +
 (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^
2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(
a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*
c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^
3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^
2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c
 - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*
b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 - ((a^2*b^5*c - 8*
a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c
^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)
*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)
*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((
b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c
*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c
^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^
3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c
^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2
))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c
- 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3
 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7
*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^3 + a^3*b*e^
5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^
2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c
^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^
3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + ((a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12
*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*
(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2
*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a
^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*
a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4
```

```
*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b
*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3
*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*
b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

maple [C] time = 0.01, size = 47, normalized size = 0.07

$$\frac{\left(\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^3 e + d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right) + x\right)}{6\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^5 c + 3\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x)
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 18.96, size = 7469, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
[Out] log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*
e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3
 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*c - b^2
)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^
4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*
e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2
*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b^5
*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3
- 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e
- 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2
```

```
*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(
2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^3*e^
3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b^5*c*d^3 + a^2*b^4*e^3
+ 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 +
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6
*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 -
48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4
- b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^
2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d
^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^
2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c
*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*
a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1
/3)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^
2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2
*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3
*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*
a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c -
 b^2)^3))^(1/3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3
*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(
-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3
*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d
^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^
2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2
*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4
+ a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) +
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a
*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^
2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^3 - 9*b^2*c^4*d^3 - 9*a*b^
3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i + 1
)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i -
 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*
e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(
1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 + 27*a*b^2*c^3*d*e^2))/12
)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*
b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2
*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
```

```
/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3)
 + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d
^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*
e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^
3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
 - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^
3 - 9*b^2*c^4*d^3 - 9*a*b^3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2
+ (2^(1/3)*(3^(1/2)*1i + 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(
2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 +
16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^
2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a
^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 +
 a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^
4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e +
 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36
 + 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e
^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3
 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
- 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2
 - 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e
^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i +
1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b
*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c
^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^
2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b
^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^
3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*c^3*x*(4*a*c - b^
2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d
^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2
*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*
c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3)
)^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 +
 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(
4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 + 1/2
)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*
c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2
*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 -
a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d
^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e)
 + (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 -
8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a
```

```
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a
*b^3*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i
- 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1
i + 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^
3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c
^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))
/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8
*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1
/3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.18 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=653

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log$$

Rubi [A] time = 1.18, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1504, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x\left(bd-ae+cdx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}}{2a}$$

$$= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\frac{\left(b+\sqrt{b^2-4ac}\right.}{2^{2/3}}}}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{}}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\,2^{2/3}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\,2^{2/3}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{b}}}}{\sqrt{3}}\right.}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.13

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{\left(cx^6 + bx^3 + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

maple [C] time = 0.01, size = 70, normalized size = 0.11

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^4 cd + (-ae + bd)\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right) + x\right)}{3a\left(2\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^5 c + \mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^2 b\right)} - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3/a*sum((c*d*_R^4+(-a*e+b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_
Z^6*c+_Z^3*b+a))-1/a*d/x
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 38.02, size = 11174, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3
*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*
b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3
+ b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^
3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a
^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c
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*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2
*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7*d^3 -
a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*
c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*
d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9
*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6 +
36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^
3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/18
+ a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2
*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5
*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 4
8*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2)))^(1/3) + log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^
3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b
^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*
e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^
4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c
*d^2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 -
 a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b
*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5
*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e -
9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*
(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^
3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3
 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b
^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*
c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3)
)^(1/3))/6 + 36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a
^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^
4*d*e^2))/18 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e
^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 +
 a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 4
8*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^
3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*
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d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c +
48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6
*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 - (2^(2/
3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^
2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) -
(27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*
d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*
a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^
2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^
2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2
*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3)
)/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/
36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^
3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^
3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*
b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*
e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e
 + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
)*(36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5
*d^3 - (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a
^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^
2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
```

```
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
))/4)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*
c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2
*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c
*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27
*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2
*c^4*d*e^2))/36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 +
1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^
2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c
*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a
^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*
b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^4*e*x*(a*e^2
+ c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6*d^3 - 10
8*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2^(2/3)*(3^(1/
2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2
 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) + (27*2^(1/
3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*d^3 - a^3
*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3
*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e
^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*
a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^
2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/12 + 10
8*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/36)*((3^(
1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3
 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e
- 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*
```

```
(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^
4*e*x*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9
*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2
+ a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e
) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(
b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 -
 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 +
3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*
c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2
*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(
1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^
2))/36)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^
2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^
2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
 - d/(a*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=655

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\ldots\right.}{\ldots}$$

Rubi [A] time = 1.11, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1504, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3
)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3
)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{2ax^2} - \frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ \cdots}}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{\cdots}}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} \cdots}}{3\sqrt[3]{2}\,a\left(b+\sqrt{b^2- \cdots}\right.}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \cdots}{3\sqrt[3]{2} \cdots}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \cdots}{3\sqrt[3]{2} \cdots}$$

$$= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2 \cdots}{\sqrt[3]{b+ \cdots}}}{\cdots}\right.}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.14

$$-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1^2b}\&\right]}{3a}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{\left(cx^6 + bx^3 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

maple [C] time = 0.01, size = 68, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^3 cd + ae - bd\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right) + x\right)}{3a\left(2\,\mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^5 c + \mathrm{RootOf}\left(\_Z^6c + \_Z^3b + a\right)^2 b\right)} - \frac{d}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+
_Z^3*b+a))-1/2/a*d/x^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 37.90, size = 13466, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
+ 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)
```

```
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 +
 b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) +
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
- 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 -
 b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) -
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3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) -
d/(2*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^
4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c
^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^
3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e
- 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
 + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 -
 36*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*
(a*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3
)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3
*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*
b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d
^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/
4)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 -
 9*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5
*d*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^
2))/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a
*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*
e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 -
 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2
- 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(
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-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b
^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^
3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*
c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2
*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*
d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e +
 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b
^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 - 3
6*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a
*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^
2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b
^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^
7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2
*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 - 9
*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5*d
*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^2)
)/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b
^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*e^
3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 1
6*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 -
48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5
*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*
c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2))
)^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^
4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e
^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*
e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27
```

```
*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 7
2*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
))/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a
^9*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b
*e - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)
^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*(
(b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3
*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*
e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5
*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3
*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a
^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e
^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/
12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2
*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3
+ 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*
a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48
*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c
*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^
2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*
d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^
2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a
^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*
a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e
 - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
```

```
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*((b
^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) +
8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d
^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e
+ 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a
^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a^6
*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2
 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/12
 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d
^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3 +
16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^
5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a
^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d
^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*
d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)x^2}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-x+\frac{x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$-\frac{x^6}{6}+\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^8*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.46, size = 37, normalized size = 0.80

$$-\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

giac [A] time = 0.42, size = 37, normalized size = 0.80

$$-\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

maple [A] time = 0.00, size = 38, normalized size = 0.83

$$-\frac{x^6}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.99, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

mupad [B] time = 0.06, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^
6/6
```

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$-\frac{x^6}{6}+\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=31

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 773, 618, 204}

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^5*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.64, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

giac [A] time = 0.58, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

maple [A] time = 0.00, size = 25, normalized size = 0.81

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x)
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

sympy [A] time = 0.12, size = 32, normalized size = 1.03

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\log\left(1-x^3+x^6\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(1-x^3\right)}{1-x^3+x^6}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^2*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.35, size = 32, normalized size = 0.82

$$
\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

giac [A] time = 0.57, size = 32, normalized size = 0.82

$$
\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$
\frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9} - \frac{\ln\left(x^6-x^3+1\right)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.95, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

3.23 $\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= \log(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
[Out] IntegrateAlgebraic[(1 - x^3)/(x*(1 - x^3 + x^6)), x]
```

fricas [A] time = 1.46, size = 34, normalized size = 0.83

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

giac [A] time = 0.59, size = 35, normalized size = 0.85

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

mupad [B] time = 1.86, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

3.24 $\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 800, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{-1+x-x^2}\right)dx,x,x^3\right)$$
$$= -\frac{1}{3x^3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x-x^2}\,dx,x,x^3\right)$$
$$= -\frac{1}{3x^3}-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-2x^3\right)$$
$$= -\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 1.45

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
[Out] -1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
[Out] IntegrateAlgebraic[(1 - x^3)/(x^4*(1 - x^3 + x^6)), x]
```

fricas [A] time = 1.06, size = 28, normalized size = 0.90

$$-\frac{2\sqrt{3}\,x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right)+3}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

maple [A] time = 0.01, size = 25, normalized size = 0.81

$$-\frac{2\sqrt{3}\ \arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^4/(x^6-x^3+1),x)
```

```
[Out] -2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
```

```
[Out] (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

sympy [A] time = 0.14, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
```

```
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=418

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.54, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((
1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3
+ I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt
[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2
^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
+ (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx &= -\frac{x^4}{4}-\frac{1}{4}\int -\frac{4x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4}+\int \frac{x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4}-\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{x^4}{4}+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3\right.}{} \\
&= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\right.}{} \\
&= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\right.}{} \\
&= -\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\right.}{9\sqrt[3]{2}\left(1\right.}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^6*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.33, size = 1036, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 18*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*s
qrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3
*arctan(sqrt(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt
(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(
6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 -
 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(
sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1
/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sq
rt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(c
os(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(
18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*
sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*c
os(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*
12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos
(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3
)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/
6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 -
 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(
2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*
log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*
12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^
(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt
(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)
*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.58, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*p
i)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i +
 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^
4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos
(2/9*pi) + sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(
3)*i + 1)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9
*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi
) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan
(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*
(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3
- 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt
```

```
(3)*sin(4/9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x
 + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9
*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*s
in(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*p
i) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)
- 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*
sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((s
qrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{x^4}{4}+\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/4*x^4+1/3*sum(1/(2*_R^5-_R^2)*_R^3*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,x^4+\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.65, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^4}{4}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^
(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(-
3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3
)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/
3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3
^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(
5/6)*1i))/36
```

sympy [A] time = 0.18, size = 31, normalized size = 0.07

$$-\frac{x^4}{4}-\mathrm{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-1458t^4+9t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 + 9*_t + x)))
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=382

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{\cdots}\right.}{3\sqrt{3}\ \cdots}$$

Rubi [A] time = 0.33, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1375, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
- I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])
^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 -
I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqr
t[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*
Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx = -\frac{x^2}{2}-\frac{1}{2}\int -\frac{2x}{1-x^3+x^6}\,dx$$
$$= -\frac{x^2}{2}+\int \frac{x}{1-x^3+x^6}\,dx$$
$$= -\frac{x^2}{2}-\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$
$$= -\frac{x^2}{2}+\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+ \ldots$$
$$= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+\ldots}}{2\sqrt{3}\ldots} \ldots$$
$$= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1\ldots\right.}\right.}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-\ldots}} \ldots$$
$$= -\frac{x^2}{2}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{\ldots}\right.}{3\sqrt{3}\ldots} \ldots$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.13

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^4*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.47, size = 1588, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/2*x^2 - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2
/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*a
rctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*s
qrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3
) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2
/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^
(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqr
t(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*
sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
 - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)
*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sq
rt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin
(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*
arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sq
rt(3) - 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) -
2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(
2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(s
qrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sq
rt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) -
2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/
3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12
*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sq
rt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^
(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(
2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin
(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))
*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*s
in(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(
3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/
3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

giac [B] time = 0.69, size = 817, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi
)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 1
0*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqr
t(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*co
s(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^
5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*
pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(
2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9
*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi)
 - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 -
sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*
cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9
*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4
/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*s
in(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*
pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sq
rt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 +
sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*
cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)
^2 + sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1)
 - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1
/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
- cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*
x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$-\frac{x^2}{2}+\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/2*x^2+1/3*sum(1/(2*_R^5-_R^2)*_R*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,x^2+\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.28, size = 309, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^2}{2}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/2 -
 (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/
6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 -
 (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

sympy [**A**] time = 0.19, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 - 27*_t**2 + x)))
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=378

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}\right.}{3\sqrt{3}\left(\frac{1}{2}\right.}$$

Rubi [A] time = 0.26, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1502, 1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx = -x+\int \frac{1}{1-x^3+x^6}\,dx$$

$$= -x-\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= -x+\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$= -x+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x \ldots}}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3} \ldots}$$

$$= -x+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1 \ldots\right.}\right.}{3\sqrt[3]{2}\sqrt{3}\left(1-i \ldots\right.}$$

$$= -x-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\ldots\right.}{3\sqrt{3} \ldots}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.12

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.39, size = 1030, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
```

```
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - x
```

giac [B] time = 0.63, size = 632, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1) - x
```

maple [C] time = 0.01, size = 41, normalized size = 0.11

$$-x+\frac{\ln \left(-\operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{6 \operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-3 \operatorname{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x+\int \frac{1}{x^{6}-x^{3}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.38, size = 330, normalized size = 0.87

$$-x+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^
(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12
)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^
(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i
+ 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [**A**] time = 0.18, size = 24, normalized size = 0.06

$$-x - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\bigl(1+i\sqrt{3}\right.}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\int}{9\ 2^{2/3}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int -}{18}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log}{}$$

$$= \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}\right.}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2
*#1^4) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[(x*(1 - x^3))/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.52, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/432*(6*1
8^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arcta
n(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan
(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18
^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3)
 + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) +
2))))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/
3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*ar
ctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqr
t(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
)^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arc
tan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(
3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)
*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1
/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(
sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 3
6*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)
*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(s
qrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqr
t(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(s
qrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/3)*x*co
s(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arc
tan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 +
 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/3)*12^(
```

```
2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3
*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(
sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/108*(18^(2/3)*12^(
1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arcta
n(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^
(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*ar
ctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*co
s(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2
))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*
arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

giac [B] time = 0.58, size = 821, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{4}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)/(x^6-x^3+1),x)
```

[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(x^3-1\right)x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)

mupad [B] time = 2.26, size = 281, normalized size = 0.68

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{12}\right)\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)

[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36

sympy [A] time = 0.18, size = 22, normalized size = 0.05

$$-\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)

[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))

3.29 $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/}\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) +
((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right.}{9\sqrt[3]{}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int}{}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\mathrm{l}}{}$$

$$= -\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right.}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^3)/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.27, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 2/27*18^(2/3)*1
2^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
 + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3)
 + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*si
n(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) +
 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(
1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sq
rt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*ar
ctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqr
t(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) +
 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(
3) + 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/216*(18^(1/3)*1
2^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(s
qrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1
/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)
*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2))
) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3
)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3)*12^(1/6)*sqrt(3)*x*
sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

giac [B] time = 0.72, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^3+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-3\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

mupad [B] time = 2.30, size = 319, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(- 3^
(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) +
3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1
i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*(3^(1/2
```

```
)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5
/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)
/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [**A**] time = 0.18, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
x)))
```

3.30 $\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right.\right.}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.28, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1504, 1374, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+$$

$$= -\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+-\frac{(-3}{}$$

$$= -\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{(3+i}{}$$

$$= -\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log(}{9\,2^{2/3}\sqrt[3]{}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^3)/(x^2*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.51, size = 1598, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*s
in(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
- 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*
12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*
x*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
- 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3
) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3
*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(s
qrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(
2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)
*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*1
2^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2
 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(
2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*
cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arct
an(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(
3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)
) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*
sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 864*cos(2/3*arcta
n(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(
3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) -
2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*a
rctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
```

```
 - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(
2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2
))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^
(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arct
an(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3
) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) +
 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3
)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt
(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))
- (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(
1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
 - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arct
an(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*
arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 108)/x
```

giac [B] time = 0.71, size = 829, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
```

```
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^5-\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)^2\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int \frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

mupad [B] time = 0.40, size = 313, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right){\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right){\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{x}-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right){\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right){\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right){\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right){\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - 1/x
 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1/3)*
3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12
 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3
^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1
i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A] time = 0.19, size = 31, normalized size = 0.07

$$-\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x))) - 1/x
```

3.31 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+\right.\right.}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Rubi [A] time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2} - \frac{1}{2}\int \frac{2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2} - \int \frac{x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{2x^2} - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\,x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \cdots$$

$$= -\frac{1}{2x^2} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \cdots$$

$$= -\frac{1}{2x^2} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \cdots$$

$$= -\frac{1}{2x^2} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log \cdots}{9\sqrt[3]{2}\cdots}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]
/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^3)/(x^3*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.35, size = 1062, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2))*log(-2*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - 8*18^(2/3)*12^(1/6)*x^2*arctan(1/216*(18^(1/3)*12^(5/6)*sq
rt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) -
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3
*arctan(sqrt(3) - 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqr
t(3) - 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/1
08*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arc
tan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3)
 - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12
^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(
sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan
(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))
/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + 4*(
18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1
/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) -
 2))^2 - 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)
*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(1
8^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/
6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt
(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*
(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3
)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) -
 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + (18^(2/3)*12^(1/6)*sqrt(3)*x^
2*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(
3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*
18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) - 2)
) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/
6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*a
rctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 +
3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 54)/x^2
```

giac [B] time = 0.64, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*s
qrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9*
pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^4 -
 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2
/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin(
1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1
/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*
cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9
*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9
```

```
*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^
4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)
^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

maple [C]   time = 0.01, size = 46, normalized size = 0.11

$$-\frac{\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{3}\ln\left(-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)+x\right)}{3\left(2\,\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{5}-\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)^{2}\right)}-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^3/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^3*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^{2}}-\int\frac{x^{3}}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

mupad [B]   time = 2.40, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)^
(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2
)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 -
3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^
(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12
)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(
2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^
(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*
log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1
/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

sympy [A]   time = 0.20, size = 32, normalized size = 0.08

$$-\operatorname{RootSum}\left(19683t^{6}+243t^{3}+1,\left(t\mapsto t\log\left(-1458t^{4}-9t+x\right)\right)\right)-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x))) - 1/(2*x**2)
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6} dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1468, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(-2+x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{-2+x}{1-x+x^2}\,dx, x, x^3\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\ &= \frac{1}{6}\log\left(1-x^3+x^6\right)+\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.03

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-2+x^3\right)}{1-x^3+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
[Out] IntegrateAlgebraic[(x^2*(-2 + x^3))/(1 - x^3 + x^6), x]
```

fricas [A] time = 1.09, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

giac [A] time = 0.46, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

maxima [A] time = 0.98, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

mupad [B] time = 1.84, size = 34, normalized size = 0.94

$$\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

sympy [A] time = 0.13, size = 37, normalized size = 1.03

$$\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

3.33 $\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx,x,x^3\right)\\
&= \log(x)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3)/(x*(1 - x^3 + x^6)), x]
```

fricas [A] time = 1.10, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.54, size = 35, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{3}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)
```

maxima [A] time = 0.99, size = 38, normalized size = 0.97

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\frac{1}{3}\,\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

mupad [B] time = 1.85, size = 36, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^3}{x-x^4+x^7}\,dx &= \int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx,x,x^3\right) \\
&= \log(x)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^3\right) \\
&= \log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right) \\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3)/(x - x^4 + x^7), x]
```

fricas [A] time = 1.58, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

giac [A] time = 0.41, size = 35, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{3}+\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^7-x^4+x),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)-1/6*ln(x^6-x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^5-2x^2}{x^6-x^3+1}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

```
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - x^4 + x^7),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

sympy [A] time = 0.14, size = 41, normalized size = 1.05

$$\log(x)-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

3.35 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=433

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right.}{2\sqrt[4]{2}}$$

Rubi [A] time = 1.13, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}}{2c}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{-b}}}{2c\sqrt{-b+\sqrt{b^2-}}}$$

$$= \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt[4]{-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)}$$

Mathematica [C] time = 0.08, size = 88, normalized size = 0.20

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&, \frac{\#1^4be\log(x-\#1)+\#1^4(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.01, size = 67, normalized size = 0.15

$$\frac{ex}{c}+\frac{\left((-be+cd)\,\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{4}-ae\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)+x\right)}{4c\left(2\,\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{7}c+\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/c*e*x+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^{4}-ae}{cx^{8}+bx^{4}+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

mupad [B] time = 9.63, size = 50213, normalized size = 115.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^
2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*
c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
```

```
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*
c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d
^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^
2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a
^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80
*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e +
61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4
*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c
^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a
^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*
```

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(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d
^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*
d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^
2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8
*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b
^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^
2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b
^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*
c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^
2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 2
0*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*
c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^
5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*
d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*
b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13
*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^
4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) -
 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^
2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c
^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^
4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^
6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d
^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16
*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*
a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5
*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*
(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2
*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*
c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^
7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 2
56*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^
4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
```

2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d -
12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 -
4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) -
(16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2
*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^
5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4
*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*
e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32
*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3
*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^
4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 -
4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e +
48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e -
200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a
*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^
3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*
a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4
*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d
^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*
e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5
*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b
^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6
*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3
*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
- 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*
b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^
2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*
a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 -
4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^

```
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(
16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^
7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*
b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^
2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*
e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c -
 b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*
c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5
*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5
 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a
^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6
*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22
*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c
^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^
2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c -
b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2
*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b
^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 +
240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*
b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^
3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d
*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2
*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*
a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^
3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d
*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^
3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4
 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3
*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^
3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^
3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) -
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 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288
*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + ata
n((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*
a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*
e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*
a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 -
120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5
*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4
 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
+ 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^
5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 +
13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3
*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*
c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 -
19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b
^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5
*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6
+ a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d
^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^
2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*
b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^
3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^
4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*
a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^
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2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*
(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^
5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c
^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*(40
96*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*
c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e -
1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*
d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
 - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*
c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2
*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 -
 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a
^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4
*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c
^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
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128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*

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d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*
c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^
3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^
3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a
*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*
c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4
*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*
c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4
*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*
c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c
^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 -
 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4
*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(163
84*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d
))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c
^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b
^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2
 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^
2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3
*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^
2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*
a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5 -
4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*
c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^
2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^
4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*
d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 +
 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 +
 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) +
 (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
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*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^
5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4)))*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
 - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
+ 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c
^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66
*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^
3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8
*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + 2*atan
((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a
^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d +
 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4
 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
- 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
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- 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a
^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^
4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4
*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*
c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 -
4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*
c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
+ 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*
c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 6
6*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a
^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
- 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a
^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*

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b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^
3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4
 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*
b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6
 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^
2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e
^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^
4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*
c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^
3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*
c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*
b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^
2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d
^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^4*
b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2
 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^
3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d
- 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a
^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4
*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3
+ 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*
a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*
e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3
*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
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6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3
/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e
^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^
5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22
*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*
b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^
4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^
4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - ((((4*x*(4096*a^4*b*c^7*d
^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*
a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
+ 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288
*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^
6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^
3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a
^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4
*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1
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/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3
- 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i
- (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^
2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a
^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^
4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d
*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 3
2*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a
^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4
- 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e +
 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e -
 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*
a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a
^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6
- 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2
*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c
^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*
d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2
*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4
 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3
*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120
*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + 2*atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2
+ 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a
^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^
6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*
a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 6
1*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a
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*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8
*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*
(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2
*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*
c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a
^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5
*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b
^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b
^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4
*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*
a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i -
 (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^
5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(1/4) + ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*
b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5
*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c
+ ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*
e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*
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c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*

```
e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*
a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 -
120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5
*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4
*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
 - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c
^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^
7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4
*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5
*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*
e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b
*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4
*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*
a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a
^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2
*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/
c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*
e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*
c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 -
 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^
2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2
*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*
b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^
5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^
4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d
^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d
^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*
b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*
c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 2
88*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (
```

(((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*

```
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e
^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a
^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 1
20*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e
^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^
3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (e*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.36 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8} dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx,x,x^4\right)\\
&= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}+\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}\\
&= \frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^4+cx^8\right)-\frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8), x]
```

fricas [A] time = 1.10, size = 216, normalized size = 3.00

$$\left[\frac{\left(b^2-4\,ac\right)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\left(b^2c-4\,ac^2\right)},\ \frac{\left(b^2-4\,ac\right)e\log\left(cx^8+bx^4+a\right)-2\,\sqrt{-b^2+4\,ac}\,(2\,cd-be)\arctan\left(-\frac{\left(2\,cx^4+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{8\left(b^2c-4\,ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a
*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^
8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 20.74, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^8+bx^4+a\right)}{8\,c}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\,c}+\frac{d\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}+\frac{e\ln\left(cx^8+bx^4+a\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b
^2)^(1/2))*d-1/4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.21, size = 3704, normalized size = 51.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] - (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) -
 (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b
^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 1
6*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*
e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16
*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^
3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2
*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*
(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) - (((
(b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4
*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) +
(32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a
*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(b*e - 2*c*d))/
(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4
*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*
(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 1
44*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 +
20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*(4*a*c - b^2)^(1/2)
) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*(64*a*c^2 - 16*b^2*c)*(
4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^3*e^4 + (b^3*(b*e -
 2*c*d)^4)/(8*(4*a*c - b^2)^2) - c^3*d^3*e - ((((b*e - 2*c*d)*(((b*e - 2*c*
d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*
a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a
```

```
*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*
a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a
*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^
2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4*d^2 + ((4*b^
2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16
*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^
2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 + 20*b^3*c*e^3
- 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(2*(64*a*c^2 - 16*b^2*c)) + 3*b*c^2*d
^2*e^2 - ((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c
^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a
*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c
^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((b*e - 2
*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
+ (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 -
 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) - 3*b^2*c*d*e^3))/(8*a^3*c^2*(4*
a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*
e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*e^3) + ((a*c - b^2)*(4*a*c - b^2)^2*(((4*b
^2*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*
a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*
b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)
) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e -
 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)
))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(8*c*(
4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - ((b*e - 2*c*d)*(((((b*e -
 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c
*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*b^2*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c -
b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(64*a*c^4*d^2 + ((4*b^2
*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e -
 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c
^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c*e^3 + 16*
a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(8*c*(4*a*c - b^2)^(1/2)) - (a*b^2*c*(4*b^
2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2)
)))/(a^3*c^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e -
8*b^3*c*d*e^3)) + ((4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*(a*b^2*e^4 - ((4*b^2
*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*
b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1
/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^
2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*
b^2*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 1
6*a*c*e)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c
^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*
a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 1
6*b^2*c)) + 24*a*b^2*c*e^3 + 16*a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(2*(64*a*c
^2 - 16*b^2*c)) + a*c^2*d^2*e^2 - ((((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512
*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(
8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)
)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64
*a*c^2 - 16*b^2*c)) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*
(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64
*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*
b*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1
/2)) + (a*b^2*(b*e - 2*c*d)^4)/(4*(4*a*c - b^2)^2) - 2*a*b*c*d*e^3))/(a^3*c
^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*
```

```
e^3)))*(b*e - 2*c*d))/(4*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 18.30, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4 + \frac{-16ac\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right) + 2ae + 4b^2\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right) - bd}{be-2cd}\right) + \left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4 + \frac{-16ac\left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right) + 2ae + 4b^2\left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right) - bd}{be-2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4
 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

3.37 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-$$

Rubi [A] time = 0.46, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1510, 298, 205, 208}

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}} -$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4a}}\right.}{2\,2^{3/4}c}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 47.55, size = 13521, normalized size = 36.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a
*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 -
48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*
a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*((2*(a*b^4
*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*
b*c^5)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b
*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^
3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)
) + ((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b
^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x - sqrt
(1/2)*((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a
*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 + (2*(a
```

*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*
a^3*b*c^5)*e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a
*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*
c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3
*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)
d^6 - 4(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^
2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*
b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^
3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 + ((a*b^6*c^4 - 12*a^2
*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4
 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a
*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4
+ 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
 (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*
a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8
- 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a
*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4
+ 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2
 - a^4*c)*e^8)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^
2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^
4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*
b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 +
 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
(a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a
^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^4*
d^6 - b*c^3*d^5*e - 5*a*c^3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2
*c^2)*d^2*e^4 + (a*b^3 + 3*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) + sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*(sqrt(1/2)*((b^2
*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*
a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 - (2*(a*b^4*c^4 -
8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*
e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)
*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(s
qrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
- 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 +
 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*
b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a
^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9
)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d
^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +

```
2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2
*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e
^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*
d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3
- 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 +
 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c
 + 4*a^4*b*c^2)*e^6 - ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^
4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e
^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d
*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*s
qrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
- 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^
6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8
)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3
 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*
e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6
*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^
6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)) + ((2*(a*b^4*c^4
- 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*
d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3
)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)) - (
(b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c
- 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x)*sqrt(sqrt(
1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*
a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/
(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))))/(c^4*d^6 - b*c^3*d^5*e - 5*a*c^
3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2*c^2)*d^2*e^4 + (a*b^3 + 3
*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3
*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 +
(a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^
6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*
b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(
a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*
c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 +
 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 +
 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 -
48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)
*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 -
6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a
^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b
^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e
^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
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 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*
d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*
b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^
5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^
2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3
*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^
3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*
c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*
c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^
6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*
c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10
)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*
e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*
d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*
a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*sq
rt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^
2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*
e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*
c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3
*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6
+ 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48
*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4
*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4
*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2
 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^
4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d
*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7
 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3
*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2
*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c
^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a
^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^
3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a
^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16
*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2
*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^
4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a
^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*
c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*
e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*
b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^
4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b
*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 +
a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c
^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3
+ (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
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a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8
*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^
2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*
a^2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*
a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3
 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3
 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3
- 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^
3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3
- 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*
(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2
*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)
*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d
^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)
*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2
*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^
4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a
^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*
c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^
3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^
2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*
c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*
a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a
^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*
a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 1
6*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^
3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^
3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*
e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^
2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^
10)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*
sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*
b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^
3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^
6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a
^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^
6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 +
48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a
^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b
^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e
^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*
d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)
*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c
^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c
^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a
^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a
*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19
```

```
*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c -
a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12
*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 +
16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b
^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*
c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48
*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
- 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^
5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^
9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*
a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*
b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3
*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3
+ a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 8.38Unable to divide, perha
ps due to rounding error%%%{-512,[0,10,0,3,5,2,7]%%%}+%%%{1152,[0,10,0,3,4,
4,6]%%%}+%%%{-512,[0,10,0,3,3,6,5]%%%}+%%%{64,[0,10,0,3,2,8,4]%%%}+%%%{1024
,[0,9,1,3,6,1,7]%%%}+%%%{-4352,[0,9,1,3,5,3,6]%%%}+%%%{512,[0,9,1,3,4,5,5]%
%%}+%%%{640,[0,9,1,3,3,7,4]%%%}+%%%{-128,[0,9,1,3,2,9,3]%%%}+%%%{-512,[0,8,
2,3,7,0,7]%%%}+%%%{7296,[0,8,2,3,6,2,6]%%%}+%%%{6144,[0,8,2,3,5,4,5]%%%}+%%
%{-4544,[0,8,2,3,4,6,4]%%%}+%%%{384,[0,8,2,3,3,8,3]%%%}+%%%{64,[0,8,2,3,2,1
0,2]%%%}+%%%{-6144,[0,7,3,3,7,1,6]%%%}+%%%{-22016,[0,7,3,3,6,3,5]%%%}+%%%{9
472,[0,7,3,3,5,5,4]%%%}+%%%{1152,[0,7,3,3,4,7,3]%%%}+%%%{-512,[0,7,3,3,3,9,
2]%%%}+%%%{2048,[0,6,4,3,8,0,6]%%%}+%%%{31232,[0,6,4,3,7,2,5]%%%}+%%%{-4352
,[0,6,4,3,6,4,4]%%%}+%%%{-8064,[0,6,4,3,5,6,3]%%%}+%%%{1792,[0,6,4,3,4,8,2]
%%%}+%%%{1048576,[0,6,0,7,9,1,8]%%%}+%%%{-3670016,[0,6,0,7,8,3,7]%%%}+%%%{2
424832,[0,6,0,7,7,5,6]%%%}+%%%{-589824,[0,6,0,7,6,7,5]%%%}+%%%{49152,[0,6,0
,7,5,9,4]%%%}+%%%{-20480,[0,5,5,3,8,1,5]%%%}+%%%{-11264,[0,5,5,3,7,3,4]%%%}
+%%%{18432,[0,5,5,3,6,5,3]%%%}+%%%{-3584,[0,5,5,3,5,7,2]%%%}+%%%{7340032,[0
,5,1,7,9,2,7]%%%}+%%%{-2097152,[0,5,1,7,8,4,6]%%%}+%%%{-1376256,[0,5,1,7,7,
6,5]%%%}+%%%{622592,[0,5,1,7,6,8,4]%%%}+%%%{-65536,[0,5,1,7,5,10,3]%%%}+%%%
{5120,[0,4,6,3,9,0,5]%%%}+%%%{18176,[0,4,6,3,8,2,4]%%%}+%%%{-23040,[0,4,6,3
,7,4,3]%%%}+%%%{4544,[0,4,6,3,6,6,2]%%%}+%%%{-8388608,[0,4,2,7,10,1,7]%%%}+
%%%{-2359296,[0,4,2,7,9,3,6]%%%}+%%%{3801088,[0,4,2,7,8,5,5]%%%}+%%%{-40960
0,[0,4,2,7,7,7,4]%%%}+%%%{-196608,[0,4,2,7,6,9,3]%%%}+%%%{32768,[0,4,2,7,5,
11,2]%%%}+%%%{-10240,[0,3,7,3,9,1,4]%%%}+%%%{17920,[0,3,7,3,8,3,3]%%%}+%%%{
-3840,[0,3,7,3,7,5,2]%%%}+%%%{4194304,[0,3,3,7,10,2,6]%%%}+%%%{2621440,[0,3
,3,7,9,4,5]%%%}+%%%{-4194304,[0,3,3,7,8,6,4]%%%}+%%%{1343488,[0,3,3,7,7,8,3
]%%%}+%%%{-131072,[0,3,3,7,6,10,2]%%%}+%%%{2048,[0,2,8,3,10,0,4]%%%}+%%%{-9
216,[0,2,8,3,9,2,3]%%%}+%%%{2176,[0,2,8,3,8,4,2]%%%}+%%%{5242880,[0,2,4,7,1
1,1,6]%%%}+%%%{-12582912,[0,2,4,7,10,3,5]%%%}+%%%{8454144,[0,2,4,7,9,5,4]%%
%}+%%%{-2195456,[0,2,4,7,8,7,3]%%%}+%%%{196608,[0,2,4,7,7,9,2]%%%}+%%%{2147
483648,[0,2,0,11,12,2,8]%%%}+%%%{-2147483648,[0,2,0,11,11,4,7]%%%}+%%%{8053
06368,[0,2,0,11,10,6,6]%%%}+%%%{-134217728,[0,2,0,11,9,8,5]%%%}+%%%{8388608
,[0,2,0,11,8,10,4]%%%}+%%%{3072,[0,1,9,3,10,1,3]%%%}+%%%{-768,[0,1,9,3,9,3,
2]%%%}+%%%{5242880,[0,1,5,7,11,2,5]%%%}+%%%{-4718592,[0,1,5,7,10,4,4]%%%}+%
%%{1376256,[0,1,5,7,9,6,3]%%%}+%%%{-131072,[0,1,5,7,8,8,2]%%%}+%%%{-2147483
648,[0,1,1,11,12,3,7]%%%}+%%%{2147483648,[0,1,1,11,11,5,6]%%%}+%%%{-8053063
68,[0,1,1,11,10,7,5]%%%}+%%%{134217728,[0,1,1,11,9,9,4]%%%}+%%%{-8388608,[0
,1,1,11,8,11,3]%%%}+%%%{-512,[0,0,10,3,11,0,3]%%%}+%%%{128,[0,0,10,3,10,2,2
```

```
]%%%}+%%%{-2097152,[0,0,6,7,12,1,5]%%%}+%%%{1835008,[0,0,6,7,11,3,4]%%%}+%%
%{-524288,[0,0,6,7,10,5,3]%%%}+%%%{49152,[0,0,6,7,9,7,2]%%%}+%%%{-214748364
8,[0,0,2,11,13,2,7]%%%}+%%%{3221225472,[0,0,2,11,12,4,6]%%%}+%%%{-187904819
2,[0,0,2,11,11,6,5]%%%}+%%%{536870912,[0,0,2,11,10,8,4]%%%}+%%%{-75497472,[
0,0,2,11,9,10,3]%%%}+%%%{4194304,[0,0,2,11,8,12,2]%%%} / %%%{1,[0,6,0,0,1,2
,2]%%%}+%%%{-2,[0,5,1,0,2,1,2]%%%}+%%%{-2,[0,5,1,0,1,3,1]%%%}+%%%{1,[0,4,2,
0,3,0,2]%%%}+%%%{6,[0,4,2,0,2,2,1]%%%}+%%%{1,[0,4,2,0,1,4,0]%%%}+%%%{-6,[0,
3,3,0,3,1,1]%%%}+%%%{-4,[0,3,3,0,2,3,0]%%%}+%%%{2,[0,2,4,0,4,0,1]%%%}+%%%{6
,[0,2,4,0,3,2,0]%%%}+%%%{-2048,[0,2,0,4,5,1,3]%%%}+%%%{512,[0,2,0,4,4,3,2]%
%%}+%%%{-4,[0,1,5,0,4,1,0]%%%}+%%%{2048,[0,1,1,4,5,2,2]%%%}+%%%{-512,[0,1,1
,4,4,4,1]%%%}+%%%{1,[0,0,6,0,5,0,0]%%%}+%%%{2048,[0,0,2,4,6,1,2]%%%}+%%%{-1
536,[0,0,2,4,5,3,1]%%%}+%%%{256,[0,0,2,4,4,5,0]%%%} Error: Bad Argument Val
ue
```

maple [C] time = 0.00, size = 51, normalized size = 0.14

$$\frac{\left(\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^6e+\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^2d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7c+4\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^4+d\right)x^2}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 9.57, size = 29445, normalized size = 78.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*
c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 1
2*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*
c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^
2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d
^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4
- a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4
+ a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4
)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*
b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3
*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*
```

```
a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*
c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d
*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*
c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 102
4*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*
b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^
4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 -
4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b
^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^
5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2
*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5
*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e
 + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*
c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*
d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128
*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 -
 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*
b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c
^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*
b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^
3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e +
6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*
b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c
^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*
c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 -
32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 3276
8*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a
^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e -
 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^
2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 +
c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 4
0*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*
d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 +
 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 +
 a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((
x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5
 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3
*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 +
 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^
3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*
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d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^
3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*
e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*
a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b
^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a
*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4
 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4
 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*
e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c
^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e
 + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*
a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256
*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c
^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e
^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 1
6384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*
b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^
3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*
1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3
*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b
^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*
c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a
^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c
^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*
e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c
^5 - 256*a^4*b^2*c^6)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 +
 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 +
 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a
*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^
3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5)
 + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*
c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^
5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c
^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^
2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^
4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e
^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^
5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*
e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^
5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*
d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 1
6*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^
2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*
a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*
b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5
*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*
a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*
c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4
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*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*
b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e
+ 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^
3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8
*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*
c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b
*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a
^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*
d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c
^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*
d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*
e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*
d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a
*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^
2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^
(1/4) - atan(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^
3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a
^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^
2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*
e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64
*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*
b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96
*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b
*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b
*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d
^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*
b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*
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c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^
3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*
e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*
a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b
^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*
a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^
2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 +
32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*
a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e
- 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a
^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^
3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*
a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^
3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 9
6*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a
*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/(
(x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^
5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^
3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5
+ 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a
^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3
*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a
^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3
*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96
*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*
b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(
a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^
4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^
4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3
*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*
c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*
e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6
*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 25
6*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*
c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*
e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e -
16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*
c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3
+ 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(
-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*
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d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i - atan(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*

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a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^
3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*
a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^
3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 9
6*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a
*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-
(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d
^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e
^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^
3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3
*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3
*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 -
6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 2
56*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6
*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4
*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e -
 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b
*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3
 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*
(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4
*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c
*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*
d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b
^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d
^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3
- 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*
a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*
a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5
*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^
3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (
-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*
d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*
e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d
^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^
3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^
3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 -
 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c -
b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4
- 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^
2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*
e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*
a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^
2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 -
32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*
b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^
6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 +
256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3
*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2
*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c
```

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 - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^
4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*
c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^
2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*
a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/((x*(4*a^3*b^3*
c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4
*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4
- 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3
*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e
^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c -
 b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c -
 b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4
 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c
^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2
*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4
*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a
^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^
5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 1024
0*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^
4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*
c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 1228
8*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2
*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 +
b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b
*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b
*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4
*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b
^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^
6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16
*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^
3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2
*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d
^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*
c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5
*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3
*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4
*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*
a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c
^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))
^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8
*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 12
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8*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3
- 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2
*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*
c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3
*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2
- 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048
*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b
^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3
+ 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3
*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*
c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a
^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a
^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d
^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 -
 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*
a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^
2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^
6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i + 2*atan(((x*(4*a^3*b^
3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c
^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^
4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c
^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2
*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e
^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5
*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d
^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16
*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 +
b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b
*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b
*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4
*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b
^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^
6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10
240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*
a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^
3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 +
 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^
4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^
```

7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4

```
- a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4
)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b
^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c
^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e
)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^
4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4
*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b
^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*
c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*
c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*
c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^
4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6
)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4
*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 25
6*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3
*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^
6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^
2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 12
8*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^
4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*c^5*d^7 + 2*a^4*c^2*
d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d
^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*
a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*
```

```
c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c
^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a
*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c
^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3
- 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

3.38 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=184

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1490, 1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)+\frac{1}{4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}$$
$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.97

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(x*(d + e*x^4))/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 1.32, size = 1535, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2
*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4
*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 -
 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4
 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*
d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
```

```
 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2
*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2
+ ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*s
qrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d -
 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

giac [B] time = 20.31, size = 1406, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^
2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^
2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*
b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*
(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c
- 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.02, size = 340, normalized size = 1.85

$$\frac{\sqrt{2}\, be \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, be \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, cd \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, cd \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, e \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, e \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^
2)*b*e-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*d+1/4*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*e-1/2*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x^2)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e x^4 + d\right)x}{c x^8 + b x^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 7.05, size = 4501, normalized size = 24.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3*
x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3
*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i + a
*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a*
c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2
*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2
*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)
/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(
a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^
3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^
3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*
d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2
- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^
2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3
```

```
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*
d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) +
2048*a^4*b*c^3*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e
^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*
a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^
3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*
b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 -
256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*
c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*
c^2 + 32*a*b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2)))*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i + atan((b^4*c
*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i + a^2*e^3*x^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3*x^2*4i - a
*b^4*d*e^2*x^2*1i + b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i - a*b*d*e^2*x^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*c*d^2*e*x^2
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*b*c^2*d^2*
e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2*b^4*e^2*(-
(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c
^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(a*b^3*e^2 +
 b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) +
 c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2
*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^3*c*d^2 - a
*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*
a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) + 2048*a^4*b*
c^3*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e^2*(-(a*b^3
*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b
*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 2
```

```
56*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^3*e^2 + b^3
*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2
*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*
b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
)))*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.39 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 0.35, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1422, 212, 208, 205}

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac} \cdots}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}} - \cdots$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}} \cdots\right.}{2\sqrt[4]{2}\sqrt[4]{c} \cdots}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^
3 + 2*c*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(a + b*x^4 + c*x^8), x]
```

fricas [B] time = 10.11, size = 13304, normalized size = 35.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*
c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5
*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2
 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c
- 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*
a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/
2)*(((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128
*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b
*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5
)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)
*x*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^
7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^
4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) + ((b^
7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 5
```

```
9*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a
^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*
b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^
3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (
a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4
*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^
4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) -
 ((b^7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^
2 - 59*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2
- 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168
*a^4*b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5
*b*c^3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^7 + ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*
a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*
b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^
4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 +
48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c
*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^
2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*
a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*
b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*
c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3
*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*
b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*sqrt((2*(
14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b
^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*
b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b
^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c
^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e +
 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*
a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4
*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 + ((a^3*b^7*c - 12*a^4*
b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^
2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b
*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)
*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^
3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^
4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d
^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8
 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
+ 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^
8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^
3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^
2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c
 + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 -
4*a^4*c)*d^2*e^6)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3
 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a
^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*
b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4
)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6
```

```
*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(
3*a^5*b*d*e^9 - a^6*e^10 + (b^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d
^9*e + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2
+ 24*a^2*b*c^3)*d^7*e^3 + 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 -
 21*(a^2*b^3*c + 2*a^3*b*c^2)*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6
+ (a^3*b^3 - 16*a^4*b*c)*d^3*e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) - sqrt(sqr
t(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/2)*(((a^
3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6
)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^
2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2
- (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*x*sqrt(
-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(
a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) - ((b^7*c^2 -
9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^
4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3
- 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e
^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3
*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^
3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3
- 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*
c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^
4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) + ((b^7*c^2 - 9*a*b^5*c^3 + 24
*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^4*c^3 + 136*a^3*
b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^
4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3 - 304*a^5*c^4)*d
^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^4 + 9*(a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*e^7 - ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5
 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64
*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a
^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4
)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 -
 a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3
)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2
)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*s
qrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c
 - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*
c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2
- a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c -
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19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*sqrt((2*(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e + 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 - ((a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)))/(3*a^5*b*d*e^9 - a^6*e^10 + (b^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d^9*e + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2 + 24*a^2*b*c^3)*d^7*e^3 + 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 - 21*(a^2*b^3*c + 2*a^3*b*c^2)*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6 + (a^3*b^3 - 16*a^4*b*c)*d^3*e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^

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5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a
^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((
10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a
*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x -
1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*
e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*
e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*
b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^
4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5
)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (
b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4
*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12
*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^
2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 -
 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))) + 1/4*sqrt(sqrt(
1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*
c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*
a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6
 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c
^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c
^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))
/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3
*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b
*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2
+ 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c
^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*
a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*
b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^
2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(
6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4
*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqr
t(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 -
 (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e -
4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)
/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c
- 8*a^4*b^2*c^2 + 16*a^5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*
a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c
^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*
a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c
^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
- 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 +
 a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c +
 a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^
3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4
*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
```

```
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*
c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3
*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 -
 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c
^3))))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.00, size = 47, normalized size = 0.13

$$\frac{\left(\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{4}e+d\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{7}c+4\,\mathrm{RootOf}\left(\_Z^{8}c+\_Z^{4}b+a\right)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4 + d}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

mupad [B] time = 8.75, size = 36707, normalized size = 97.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3
```

```
*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 4
0*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*
b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*
c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*
c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 4
9152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b
*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 +
 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*
a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^
5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a
*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*
d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2
*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3
*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 1
6*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 -
16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e -
16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*
d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3
 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 +
 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^
3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c
^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^
5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*
d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e
^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d
*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^
3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^
3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a
^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 1
28*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
 - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^
2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2
*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^
5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^
5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e +
 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4
*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2
 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 819
2*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*
c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*
```

a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^
3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a
^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c
^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c -
16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
- 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
- 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^
3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e
^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2
+ 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*
d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3
*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*
b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^
4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2
*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3
*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*
a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*
c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))
^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d
^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8
192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^
4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*
c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40
*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b
*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
- 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^
5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*
e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*
c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2
*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e
^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^
4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b
^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^

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3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*
c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c
^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c
^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 -
8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^
2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)
))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*
e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a
^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^
2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a
^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a
^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^
4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^
2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 81
92*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4
*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c
^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*
a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*
c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
- 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
 - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c
^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*
e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^
2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4
*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^
3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3
*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c
^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^
2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^
3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2
*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))
)^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 1
1*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8
*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 12
8*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
- 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2
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*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*
c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5
*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*2i - atan((((-(b^7*c*d^4 + a^3*b^5*e^4
- a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*
c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*
e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(
((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c
^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*
c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4
*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*
b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^
2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3
- 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608
*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*
c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 1126
4*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3
*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e
- 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b
^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^
4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4
 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*
e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c
^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2
 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*
a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(
1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256
*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 -
192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3
*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*
b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*
d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e
^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6
*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*
b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(
1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*
c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e
 + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*
a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*
e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2
+ 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^
4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5
```

*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^
3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c
^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^
3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*
d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e
^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^
3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 1966
08*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^
2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11
264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b
^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*
e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-
(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*
d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e
^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^
3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3
*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e
^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 +
6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 2
56*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
- 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d
^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*
a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^
7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2
*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c
^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*
a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^
6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3
*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 6
4*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^
2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*
e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^
4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e
^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*
b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*
d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7
*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/
4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b
^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*
b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4
*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*
a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*
c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d
*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*
c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 19
6608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*
b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 -
11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2
*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*
d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*

```
(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^
2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c
*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*
d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b
^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2
*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
+ 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^
5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6
*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 6
4*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*
c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d
^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b
*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 1
6*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^
5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*
b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d
^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*
c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e
^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
+ b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^
5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b
^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*
c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a
^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c
*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*
e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c
^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196
608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b
^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 1
1264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*
b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d
*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(
-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2
*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*
e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d
^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^
3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*
e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 +
 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*
d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64
*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c
^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^
```

2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*2i - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +

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16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*
d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^
6*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^
5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*
b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d
^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*
c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^
4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
+ a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 -
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*
b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d -
 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152
*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5
*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e +
20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4
+ a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 +
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*
c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*
e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1
i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 -
16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^
4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 25
6*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e
^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d
^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c
^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))
*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c
^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*
c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4
*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*
b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^
2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3
- 256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^
4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b
^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d
^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
```

```
4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*
a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^
3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*
c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c
^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c
^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 -
8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^
2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)
))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4
096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7
*c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2
 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024
*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^
4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*
b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^
4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2
*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3
*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*
a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*
c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))
^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*
d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*
b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*
e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a
^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b
^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e -
16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d
^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7
*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4
- 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e
- 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2
*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a
^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^
3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*
```

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b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6
*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 20
48*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c
*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 1
92*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*
e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b
^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^
7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2
*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c
^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*
a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^
6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3
*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 6
4*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^
2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e
^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
- b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
) - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
 + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^
7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2
- 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2
*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4
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*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*
c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^
5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*
a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c
^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*
c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 -
16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^
3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c
*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b
^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^
3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c
^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4
 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
 a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4
*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3
*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^
5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*
(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b
^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d -
262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*
a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*
c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 2
0480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 -
 a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^
2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c
*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e
 + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
+ a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i
 - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 1
6*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4
*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256
*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e^
6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^
4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^
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5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*
(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^
2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c
*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*
d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b
^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2
*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
+ 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
 256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4
 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^
4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^
2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a
^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3
*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c
^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^
2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^
3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2
*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))
)^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 40
96*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*
c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2
- 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*
a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4
*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b
^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4
*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*
e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*
d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a
^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c
^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^
(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d
*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b
*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e
^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^
3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^
2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 1
6*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^
2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a
^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*
a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b
^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^
2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b
^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
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b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*
c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*
d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^
6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3
*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b
^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*
d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 204
8*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*
d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 4
8*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16
*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 1
28*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^
2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 12
8*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*
b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 19
2*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e
^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^
3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7
*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*
e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^
6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a
*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6
*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*
e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64
*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2
 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e^
4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.40 $\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)} dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^4\right)\\
&= \frac{d\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{d\log(x)}{a}-\frac{d\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}+\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\operatorname{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)), x]
```

fricas [A] time = 2.51, size = 240, normalized size = 3.08

$$\left[-\frac{\left(b^2-4ac\right)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4ac\right)d\log(x)+\sqrt{b^2-4ac}\,(bd-2ae)\log\left(\frac{2c^2x^8+2bcx^4+b^2-2ac-\left(2cx^4+b\right)\sqrt{b^2-4ac}}{cx^8+bx^4+a}\right)}{8\left(ab^2-4a^2c\right)}, -\frac{\left(b^2-4ac\right)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4ac\right)d\log(x)-2\sqrt{-b^2+4ac}\,(bd-2ae)\arctan\left(-\frac{\left(2cx^4+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{8\left(ab^2-4a^2c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c -
(2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), -
```

```
1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 20.63, size = 78, normalized size = 1.00

$$-\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arct
an((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\;a}+\frac{e\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}+\frac{d\ln(x)}{a}-\frac{d\ln\left(cx^8+bx^4+a\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/a*d*ln(x)-1/8*d*ln(c*x^8+b*x^4+a)/a+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4
+b)/(4*a*c-b^2)^(1/2))*e-1/4/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-
b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.27, size = 8454, normalized size = 108.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 -
 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b*c
^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*
(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)
)/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c
^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*
c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
```

```
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 -
 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*
e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1
280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a
*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 460
8*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^
5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1
/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*
c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*
a*e - b*d)^3)/(1024*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)))*(2*a*e
 - b*d))/(8*a*(4*a*c - b^2)^(1/2)) - ((((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*
d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 6
4*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c
 - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a
*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*
a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^
2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 -
64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*
a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d -
 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 46
08*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c
^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^
2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*
a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c
^5)*(2*a*e - b*d)^4)/(8192*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2))*(5*b
^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e)
)/(32*a^5*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)) - ((((4*b^2*d
 - 16*a*c*d)*(((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b
^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c
 - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 -
64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^3)/(1024*a^3*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((
4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 1
6*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b
^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) +
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c
^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*
b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))
))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b
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^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2
*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e
))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*
c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b
*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*
e - b*d)*(11*b*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 1
6*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5
*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*
c^4*e^3 + 108*b*c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(8*a*(4*a*c - b^2)^
(1/2)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d
 - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 5
76*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)
) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(1
6*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c
- b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*
e - b*d)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 -
64*a^2*c)) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*
b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024
*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*
a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*
a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*
c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4
*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 8
64*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c
- b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) - ((1280*b^5*c^4 - 4608*a*b^3*c^5
)*(2*a*e - b*d)^5)/(32768*a^5*(4*a*c - b^2)^(5/2)))*(144*a^3*c^3*d - 40*b^6
*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*d - 40*a^2*b^3*c*e + 40*a^
3*b*c^2*e))/(256*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 20*b^2*d^2 + 81*a*c
*d^2 - a*b*d*e)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a
^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3) + (4*(4*a*c -
b^2)^(5/2)*(5*b^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d +
 3*a^2*b^2*c*e)*(c^4*d*e^4 + ((4*b^2*d - 16*a*c*d)*(a*c^4*e^4 + ((4*b^2*d -
 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 2
56*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)
))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*
a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*
a^2*c)) - 16*b*c^4*d*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c
*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128
*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)
^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a
^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*
d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e -
b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d)
)/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d
- 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^
4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64
*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/
(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d)
)/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d
)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*
d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*
c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2
)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d
)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(
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16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b
^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*
e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256
*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))
/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*
b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(
1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((2*a*e - b*d)*(((2*a*e -
 b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^
2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64
*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^
2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/
(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^3)/
(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(8*a*(4*a*c - b^2)^(1/2
)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^4)/(32*a^3*(16*a*b^2 - 64*
a^2*c)*(4*a*c - b^2)^2)))/(c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e
)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*
e - 32*a^3*b*c^4*d*e^3)) + ((4*a*c - b^2)^2*(((4*b^2*d - 16*a*c*d)*(((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*
a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/
(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))
/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) +
((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 6
4*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2)))
)/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d
- 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c
^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4
*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4
*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 -
 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(
2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b
^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2
- 64*a^2*c)*(4*a*c - b^2))))/(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d
- 16*a*c*d)*(2*a*e - b*d)^3)/(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/
2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(
((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d -
 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4
*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(
1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e
- b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d -
 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*
c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e -
b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e
 + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^
2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1
/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2))
+ ((2*a*e - b*d)*(a*c^4*e^4 + ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*
(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4
*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*
a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*
e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - 16*b*c^4*d*e^3))/(8*a*(4
*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*e - b*d)^5)/(128*a^4*(4*a*c - b^2)^(5/2)
))*(144*a^3*c^3*d - 40*b^6*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*
d - 40*a^2*b^3*c*e + 40*a^3*b*c^2*e))/(2*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c
```

```
*d^2 - a*b*d*e)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*
a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3)))*(2*a*e - b*d))/(4*a*(4*a*c - b^2)^(
1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-\cdots}}$$

Rubi [A] time = 0.68, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1504, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax}-\frac{\int \frac{x^2\left(bd-ae+cdx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{d}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}}{2a}$$

$$= -\frac{d}{ax}+\frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,a}-\frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{-b}}}{2\sqrt{2}\,a}$$

$$= -\frac{d}{ax}-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}}{\sqrt[4]{-b+\sqrt{b^2}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.06, size = 85, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**C**] time = 0.01, size = 72, normalized size = 0.18

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^6cd+(-ae+bd)\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{4a\left(2\,\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7c+\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3b\right)}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/a*d/x-1/4/a*sum((_R^6*c*d+(-a*e+b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 9.46, size = 39028, normalized size = 99.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)
```

```
[Out] atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*
b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^
3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^
3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a
*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 2
40*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48
*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*
c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2
*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*
```

```
c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1
024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81
920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 327
68*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*
a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*
b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5
*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e
^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*
e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d
^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^
8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*
c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3
*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6
*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3
*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 -
 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^
4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*
c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))
^(1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*
a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a
^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b
*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8
*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(
4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b
^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3
*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^
2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 5
1200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10
240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048
*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096
*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b
^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5
*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^
4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3
*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^1
1*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32
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*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^
12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13
*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^
3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b
^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2
*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e +
 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^
4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*
c^3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288
*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*
a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^
9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*
a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
- 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^
2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e -
 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128
*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17
*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*
c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c
^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*
d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3
+ 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*
a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^
14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*
b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^
14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*
(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^
5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
+ 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 4
4*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*
c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 -
 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c
^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d
^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
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320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a
^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)
^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*
e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*
d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^
3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e -
288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096
*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^
3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*
a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^
16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^
5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4
*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d
*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*
e) - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 28
16*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256
*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^
12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576
*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) +
 x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d
*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e
^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2
+ 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 +
 a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a
^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a
^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4
 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^
4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^
3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*
c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2
 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 -
6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*
e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4
+ 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a
^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^
4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*
e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2
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*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*
b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3
- 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28
8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + at
an((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^
2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^
5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*
c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c
^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2
*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8
*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a
^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^
5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
- 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*
d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d
*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^
2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102
4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192
0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768
*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^
14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*b*
c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d
^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3
 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e
- 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2
*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*
d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^
7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c
^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d
^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*
a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 12
8*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
 - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3
*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*
b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^
3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^
6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*
a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
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) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3
*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c
^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8*d
^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2
- 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 512
00*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1024
0*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a
^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a
^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6
*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c
^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*
d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c
^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*
b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a
^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12
*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b
*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^
4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*
e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8
*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e
^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 4
0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*
b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d
*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 2
00*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a
^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^
9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
```

120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c
^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^
6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5
*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*
e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 +
4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^
12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14
*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^
7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14
*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4
*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5
- 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 +
4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*
a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^
4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*
e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*
b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3
- 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28
8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a
^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
- b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 20
0*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^
5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16
*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*
d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e

^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e
 - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)
 - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816
*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a
^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12
*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a
^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x
*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 -
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^
3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 +
6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^1
2*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^
3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*
a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - 2*at
an((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^
2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^
5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*
c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c
^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2
*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8
*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a
^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^
5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3
e(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*
d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d
*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c

- b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^
2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102
4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192
0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768
*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^
14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16
*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^
5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*
e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2
*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*
d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*
b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a
^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12
*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b
*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^
4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*
e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8
*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e
^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 4
0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*
b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16
*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^
8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4
d^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6
*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a
^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*
b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^
3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d
^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 +
51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1
0240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 204
8*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i -
4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^
12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14
*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^
7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14
*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x

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*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
- 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c
*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3
*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*
b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a
^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e
^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e -
 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 +
256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(409
6*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288
*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*
a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a
^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c
^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^
4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*
d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d
*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3
- 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 -
 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 76
8*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 2
4576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^
2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^1
4*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^
6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d
^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^
5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3
- 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*
c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b
^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240
*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a
```

^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*

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c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a
*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d
^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e
 + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*
a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^
2*c^3)))^(1/4) - 2*atan((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^
3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a
^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*
d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*
e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8
 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(
x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e
^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c
^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2
*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*
d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^
2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^
2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 327
68*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^
14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^1
5*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b
^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^1
5*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d
^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^
3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e +
 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2
+ 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d
*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*
a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2
*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^
7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4
*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e
^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b
^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*
a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4
8*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5
*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^
2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4
*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
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128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^
3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2
 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a
^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2
*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128
*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
 - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b
^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3
*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1
/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 1228
8*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024
*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304
*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^
15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*
b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b
^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^
7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c
^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*
b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^
5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32
*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*
a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
```

d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 122

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88*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 102
4*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 9830
4*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a
^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*
c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c
^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*
e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d
^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^
5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5
*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11
*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^1
1*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^
2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16
*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c
^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^1
1*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^1
3*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 +
 a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6
*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a
^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 -
 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*
b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3
*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^
2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7
*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - d/(a*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)

[Out] Timed out
```

3.42 $\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Rubi [A] time = 0.31, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1490, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^
2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{d}{2ax^2} - \frac{\text{Subst}\left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Sub}}{}$$
$$= -\frac{d}{2ax^2} - \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4a}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.45

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^6c+\#1^2b}\&\right]}{4a}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)), x]
```

fricas [B] time = 2.21, size = 2772, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2
- 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d
^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 -
 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2
 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5
*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (
```

```
a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*
c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^
2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a
^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2
*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b
^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*
c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3
 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
+ sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3
*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*
e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13
*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 -
4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*
c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*
(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3
*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b
^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*
a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*
c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 -
 a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*
c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*
b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
- a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2
*d)/(a*x^2)
```

giac [B] time = 22.52, size = 3006, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*
c^3)*d*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

```
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*
a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2
*c^2)*abs(a)*e + (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2)*d - (2*a^2*b^3*c^2 - 8
*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2
- 4*a*c)*a^2*b*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 - 4*
a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*
b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) - 1/8*((sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3
- 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a) - (2*a*b
^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c
)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*
a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^
3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(a)*e - (2*a*b^4*
c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*e)*arc
```

```
tan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^2*b^4
- 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^
3*c^3)*abs(a)*abs(c)) - 1/2*d/(a*x^2)
```

maple [B] time = 0.02, size = 365, normalized size = 1.83

$$\frac{\sqrt{2}\, bcd \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}+\frac{\sqrt{2}\, bcd \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, ce \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, ce \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, cd \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{\sqrt{2}\, cd \arctan\left(\frac{\sqrt{2}\, c x^2}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\, a}-\frac{d}{2a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x^2)*e+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*d-1/4/a*c*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x^2)*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*e+1/4/a*
c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b*d-1/2/a*d/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2
)
```

mupad [B] time = 7.62, size = 15013, normalized size = 75.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^
12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*
c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e -
8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^
2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
```

```
 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e +
4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a
^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7
*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e
 - 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2
))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d
^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*
(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*
d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*
e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i
- ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
 - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^
3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6
*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d
- 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a
^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^
2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d
*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 307
2*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a
^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^
7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*
a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(
b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d
*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 6
4*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a
^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^1
1*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^
3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 +
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(
b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2
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*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d
*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a
*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*
d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b
*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 -
 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024
*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^
4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*
e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4
096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11
*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^
2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^
2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a
^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b
^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^
2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*
b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*
c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*
e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16
*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 +
 a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*
b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^
(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^
2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*
a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^
4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32
*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^
13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7
*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e +
 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*
b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*
d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e
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) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^
8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c
^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^
2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b
^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
+ 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d
^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e
 - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a
^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c
^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e
^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e
^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
 atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*
a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^
4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32
*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 -
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^1
2*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c
^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8
192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2
 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
- 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4
096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^
12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*
d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e -
 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2)
)*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^
4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(
8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d
^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4

```
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i -
 ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*
c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d -
 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^
12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2
*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2
) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072
*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^
12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7
*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a
^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64
*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^
10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11
*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3
 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*
b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d
*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
- 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*
c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 -
32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024*
a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4
*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
```

```
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 40
96*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*
b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2
*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2
 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^
11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^
3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2
 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b
^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c
^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*e
^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*
a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^
3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 +
a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(
1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4
*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 1
6*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^1
3*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*
c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e +
16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^
2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b
*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d
^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e)
 - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8
*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^
5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2
*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^
3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(
-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) +
 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^
4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e
- 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a^
9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^
6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*
a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^
2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e^
```

```
2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
d/(2*a*x^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

3.43 $\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=394

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right.}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)\cdots}$$

Rubi [A] time = 0.63, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1504, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(
-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{3ax^3}-\frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{d}{3ax^3}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+\cdots}}{2a}$$

$$= -\frac{d}{3ax^3}+\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\sqrt{-b-\cdots}}}{2a\sqrt{-b-\sqrt{b^2\cdots}}}$$

$$= -\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b+\cdots}}\right)}{2\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right)\cdots}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]+\frac{4d}{x^3}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e
*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.01, size = 68, normalized size = 0.17

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^4cd+ae-bd\right)\ln\left(-\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)+x\right)}{4a\left(2\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^7c+\operatorname{RootOf}\left(\_Z^8c+\_Z^4b+a\right)^3b\right)}-\frac{d}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
[Out] -1/3/a*d/x^3+1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 10.22, size = 65350, normalized size = 165.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
[Out] atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86
*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^
3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^
5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3
- 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3
*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^
4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*
e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^
3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^
```

```
4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
- 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3
*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c
*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 29
2*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^
5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b
^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c
^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d -
 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 19660
8*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a
^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13
*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14
*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*
c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^
2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^
5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13
*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^
3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^
13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^
2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c
^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*
c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3
*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7
*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
```

/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +

96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c
^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16
*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^
11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 2
56*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^
11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a
^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*
a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^1
0*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*
b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a
^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*
b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231
*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/
2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*
d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*
d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a
^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*
b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4)*1i)/(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*
d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^
3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e +
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*
e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*
a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^
6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*
a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*
a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^
4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*
b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a
^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
d^4(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56
*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*
e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 +
320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a
^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 25
6*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262

144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*

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b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^
3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e
- 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 1
6*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d
*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
+ 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 +
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(2
62144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15
*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c
^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d
^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2
+ 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2
- 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 20
48*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 16
3840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a
^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d
^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*
d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 +
6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48
*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c
^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d
^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^
3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*
b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b
^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a
^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d
^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 +
 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*
e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e -
 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) -
 x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e
^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 -
 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 2
4*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*
d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^
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4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + atan((((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^

```
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^1
1*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^
14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^1
5*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6
*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4
+ a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a
^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b
^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c
^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68
0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a
^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e -
 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*
c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*
b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*
b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b
*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6
 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*
c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b
^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d
^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4
+ a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e
- 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^
3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*
a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2
*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3
*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^
3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^
2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c
^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 1
6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11*d^4
+ a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a
^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b
^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c -
```

b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c
^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68
0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a
^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
c^3)))^(1/4)(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d
*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3
*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e -
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^
2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6
*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e
 - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2
00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a
^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^
8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^
7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e +
262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 102
4*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 1228
80*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 4096
0*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^1
3*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11
*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d
^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*
a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*
b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^
4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*
a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*
a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16
*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a
^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^1
0*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9
*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a
^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*

c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3
*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11
*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d
^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*
a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*
b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*
c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
+ a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3
*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a
^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^
4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*
d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*
e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 -
 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*
c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^
13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b
^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6
*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2
+ 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 -
 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 +
 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 2457
6*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-
(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*
d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1

/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*
c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c
- b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4
+ 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366
*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b
^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^
2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e
^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^
3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 25
6*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c
^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 +
128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2
+ 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4
+ 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e -
128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^1
0*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 +
4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9
*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*
b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*
c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 +
280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*
c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2
*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b
^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2
*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2
) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e -
640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320
*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b
*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^
11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^
11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4
- 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2
*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b
^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 +
6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^
4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e
^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3
+ 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b
^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^

7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^
4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d
^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6
*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*
a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^
2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^
3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*
a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16
*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c
^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^
2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2
 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2
 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24
576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*
(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12
*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5
 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^
2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e
^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e
- 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a
^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4
+ 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3
 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a
^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^
7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^
6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4
+ 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^
9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a
^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5
*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d
^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a
^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e
- 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 3

20*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3
*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*
a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b
^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^
4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^
2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c -
b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 +
6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a
^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(
4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*
e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3
+ 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*
b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*
a^10*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3
*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
- 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4
+ a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*
a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*
b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*
c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*
(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 6
80*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*
a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^
2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^
5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e +
49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i + x
*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 133
12*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 10
24*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 655

```
36*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a
^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6
*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a
^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a
^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4
 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b
^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^
5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*
a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(
1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e
 - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 +
320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^
3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256
*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i +
64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*
d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64
*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3
+ 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e +
48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 6
4*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*
c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 +
 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^
4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^1
1*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1
/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4
*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a
^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e
+ 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b
^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3
*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16
*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4*b^7*e^4 +
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
```

256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
 - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d
^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c
^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c
^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*
d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 +
 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^
7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2
*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5
*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 -
a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e -
128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*
c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*
b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e
^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c -

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b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e
 + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*
b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*
c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3
*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8
*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^11*d^4 + a^4
*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^
5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8
*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a
^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^
2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^
2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a
*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4
*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*
c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
+ 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^
6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2
*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*
e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78
*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^
5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*
a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880
*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*
a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*
b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d
^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 1
1*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^
2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^
5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
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680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 48
0*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d
^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128
*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96
*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 1
6*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*
a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10
*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4
*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*
b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(
b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d
^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c
^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c -
 b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4
+ 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*
a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^
3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2
*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^
3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3
*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256
*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*
c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*
a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*
d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)
^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10
*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 +
 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 4
8*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*
c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*
```

d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e
^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4
*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 409
6*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^
16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3
*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c
^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^
6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^
5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e
 + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d
e))(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2
)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a
^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-
(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*
c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c
^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 -
4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b
^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*
c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
- 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c
- b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c
^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 -
192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2
*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^
7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b
*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^
8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^
7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c
^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*
c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*
d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7
*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*
e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e
^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b
^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a
^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^
2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^
2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4
*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*
d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7
*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(
1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 +
 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d
^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6

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*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*
b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e
^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*
a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5
*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 -
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
 - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
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^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d
^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c
^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c
^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*
d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 +
 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^
7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2
*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5
*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 +
a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e -
128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*
c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*
b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e
^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e
 + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*
b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*
c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3
*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8
*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4
*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^
5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8
*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a
^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
 - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^
2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^
2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a
*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4
*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*
c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 1
12*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
+ 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*
d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^
6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2
*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2
```

e^2(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*
a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*
e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78
*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^
5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*
a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880
*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*
a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*
b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d
^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 1
1*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^
2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^
5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 48
0*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d
^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128
*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96
*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 1
6*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*
a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10
*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4
*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*
b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
- 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^1
1*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4

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- 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*
d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^
2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6
*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4
*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d
*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^
2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^
10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7
*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^
3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*
a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a
^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2
*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3
*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
- 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a
^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*
b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^
6*e + 262144*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*
e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d
^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e
^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e +
24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e)
)*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*
b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*
a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2
*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a
^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*
c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2
*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 1
8*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
- 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192
*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^
6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d
^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^
```

7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d
^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e
^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*
d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7
*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e
^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^
4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4
 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3
+ 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*
c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*
b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 -
 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 -
 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a
*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^
2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^
3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2
*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
+ 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^
8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4
)*1i + ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*
a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c -
b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 8
6*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4
*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b
^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b
^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3
 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^
3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b
^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c
*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a
c - b^2)^5)^(1/2))/(512(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3
*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*
c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d
- 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 1966
08*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 491
52*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*

a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*
a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*
b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^1
5*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d
*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3
*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e -
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^
2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6
*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e
 - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2
00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a
^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b
*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 -
16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*
a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 -
 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*
a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x
*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2
 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 1
6*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*
a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^
4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11
*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5
)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2
*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5
*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480
*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b
^2*c^3)))^(1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*
c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
+ a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3
*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a
^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^
4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*
d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*
e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 -
 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c

```
- b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*
c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - d/(3*a*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

3.44 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=278

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+\right.}{4\sqrt{6}}$$

Rubi [A] time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1502, 1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4\left(1-x^4\right)}{1-x^4+x^8}\,dx = -x+\int \frac{1}{1-x^4+x^8}\,dx$$

$$= -x+\frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -x+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\ldots$$

$$= -x-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\ldots$$

$$= -x-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\ldots$$

$$= -x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\ldots$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(1-x^4\right)}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(1 - x^4))/(1 - x^4 + x^8), x]
```

fricas [A] time = 1.27, size = 218, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
 x
```

giac [A] time = 0.45, size = 208, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

maple [C] time = 0.01, size = 34, normalized size = 0.12

$$-x+\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+3\,\mathrm{RootOf}\left(9\_Z^4+1\right)x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x+\int \frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.92, size = 56, normalized size = 0.20

$$-x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/
12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/
12)
```

sympy [A] time = 0.23, size = 170, normalized size = 0.61

$$-x-\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}-\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+2\sqrt{6}\,x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}\,x^3+3x^2-\sqrt{6}\,x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}\,x^3+3x^2+\sqrt{6}\,x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2
*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*
x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*
x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(
6)*x + 1)/24
```

3.45 $\int \frac{x^3(1-x^4)}{1-x^4+x^8} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)$$
$$= -\frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[(x^3*(1 - x^4))/(1 - x^4 + x^8), x]
```

fricas [A] time = 0.73, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

giac [A] time = 0.63, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right) - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 1.51, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

sympy [A] time = 0.15, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

3.46 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2\right.$$

Rubi [A] time = 0.29, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}\ \ldots}{1+\sqrt{2+\ldots}}}{4\sqrt{3\left(2\ldots\right.}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}\ \ldots$$

$$= \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\ldots$$

$$= \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\ldots$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2
*#1^5) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(1-x^4\right)}{1-x^4+x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(1 - x^4))/(1 - x^4 + x^8), x]
```

fricas [B] time = 1.11, size = 715, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.48, size = 253, normalized size = 0.71

$$-\frac{1}{24} \left(\sqrt{6}+3 \sqrt{2}\right) \arctan\left(\frac{4 x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24} \left(\sqrt{6}+3 \sqrt{2}\right) \arctan\left(\frac{4 x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24} \left(\sqrt{6}-3 \sqrt{2}\right) \arctan\left(\frac{4 x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24} \left(\sqrt{6}-3 \sqrt{2}\right) \arctan\left(\frac{4 x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48} \left(\sqrt{6}+3 \sqrt{2}\right) \log\left(x^2+\frac{1}{2} x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48} \left(\sqrt{6}+3 \sqrt{2}\right) \log\left(x^2-\frac{1}{2} x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48} \left(\sqrt{6}-3 \sqrt{2}\right) \log\left(x^2+\frac{1}{2} x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48} \left(\sqrt{6}-3 \sqrt{2}\right) \log\left(x^2-\frac{1}{2} x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
```

```
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1)
```

maple [C] time = 0.01, size = 46, normalized size = 0.13

$$-\frac{\left(\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^6-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^2\right)\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(x^4-1\right)x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.99, size = 248, normalized size = 0.70

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(-1+\sqrt{3}\,1i\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(-1+\sqrt{3}\,1i\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(-1+\sqrt{3}\,1i\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(-1+\sqrt{3}\,1i\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x}{2\left(1+\sqrt{3}\,1i\right)^{3/4}}-\frac{2^{3/4}\sqrt{3}\,x\,1i}{2\left(1+\sqrt{3}\,1i\right)^{3/4}}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}1i}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\,1i}{2\left(1+\sqrt{3}\,1i\right)^{3/4}}+\frac{2^{3/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1i\right)^{3/4}}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) - (3
^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/
4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4)) - (2^
(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/
12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)) + (2^(
3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.16, size = 27, normalized size = 0.08

$$-\mathrm{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(442368t^7-384t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 3
84*_t**3 + x)))
```

3.47 $\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx$

Optimal. Leaf size=50

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1490, 1164, 628}

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1-x^4\right)}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\
&= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.88

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)-\log\left(-x^4+\sqrt{3}x^2-1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-x^4\right)}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] IntegrateAlgebraic[(x*(1 - x^4))/(1 - x^4 + x^8), x]
```

fricas [A] time = 0.87, size = 41, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4+2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))
```

giac [A] time = 0.44, size = 31, normalized size = 0.62

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

maple [A] time = 0.02, size = 39, normalized size = 0.78

$$-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+1)/(x^8-x^4+1),x)
[Out] -1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(x^4-1\right)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.89, size = 20, normalized size = 0.40

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,x^2}{x^4+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

sympy [A] time = 0.13, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12
```

3.48 $\int \frac{1-x^4}{1-x^4+x^8} \, dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(\right.$$

Rubi [A] time = 0.22, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3]
)]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right.}}{1+\sqrt{2+}}}{4\sqrt{3\left(2\right.}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{}$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(1 - x^4 + x^8), x]
```

fricas [B] time = 0.69, size = 715, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.54, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$\frac{\left(-\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{4}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)+x\right)}{8\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{7}-4\operatorname{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.00, size = 208, normalized size = 0.59

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.23, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(9216t^{5}-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

3.49 $\int \frac{1-x^4}{x\left(1-x^4+x^8\right)} \, dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\ &= \log(x)-\frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\ &= \log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\ &= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\ &= \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
[Out] IntegrateAlgebraic[(1 - x^4)/(x*(1 - x^4 + x^8)), x]
```

fricas [A] time = 0.89, size = 34, normalized size = 0.83

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

giac [A] time = 0.45, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}+\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

mupad [B] time = 1.89, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log(x)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

3.50 $\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=280

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.21, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1504, 1372, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{}}}{1+\sqrt{}}}{4\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}\,x+x}}{4\sqrt{6}}$$

$$= -\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x\right.}{4\sqrt{6}}$$

$$= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{}}}\right)}{2\sqrt{6}}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(x^2*(1 - x^4 + x^8)), x]
```

fricas [A] time = 0.99, size = 224, normalized size = 0.80

$$\frac{4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\sqrt{3}\sqrt{2}\,x\log\left(x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-\sqrt{3}\sqrt{2}\,x\log\left(x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1\right)-24}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x
^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2
 - 2)) + 4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqr
t(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*
x^2 - 2)) + sqrt(3)*sqrt(2)*x*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
 1) - sqrt(3)*sqrt(2)*x*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
24)/x
```

giac [A] time = 0.58, size = 210, normalized size = 0.75

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

maple [C] time = 0.01, size = 38, normalized size = 0.14

$$-\frac{\mathrm{RootOf}\left(9\_Z^4+1\right)\ln\left(9\,\mathrm{RootOf}\left(9\_Z^4+1\right)^3 x-3\,\mathrm{RootOf}\left(9\_Z^4+1\right)^2+x^2\right)}{4}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int \frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

mupad [B] time = 1.86, size = 58, normalized size = 0.21

$$-\frac{1}{x}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
1/x
```

sympy [A] time = 0.23, size = 168, normalized size = 0.60

$$-\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}-\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
 sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
 1)/24 - 1/x
```

3.51 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1490, 1281, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,x,x^2\right)-\\
&= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\right.\\
&= -\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1-\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.55

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &
 ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(x^3*(1 - x^4 + x^8)), x]
```

fricas [B] time = 1.03, size = 188, normalized size = 2.11

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}x^2\log\left(2x^4+\sqrt{6}\sqrt{2}x^2+2\right)-\sqrt{6}\sqrt{2}x^2\log\left(2x^4-\sqrt{6}\sqrt{2}x^2+2\right)-24}{48x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^2*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^2*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24)/x^2
```

giac [A] time = 0.53, size = 81, normalized size = 0.91

$$-\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right)-\frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2 -
sqrt(3)) - 1/2/x^2
```

maple [A] time = 0.01, size = 70, normalized size = 0.79

$$-\frac{\arctan\left(2x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x^2+\sqrt{3}\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\,\ln\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*l
n(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

mupad [B] time = 0.10, size = 56, normalized size = 0.63

$$\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

sympy [A] time = 0.23, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2)
```

3.52 $\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}$$

Rubi [A] time = 0.27, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{1}{3}\int \frac{3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}\,x+x^2}}{8\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x\right.}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\ \cdots}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right.}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]
/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^4)/(x^4*(1 - x^4 + x^8)), x]
```

fricas [B] time = 1.19, size = 608, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqr
```

```
t(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) +
8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)
*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 4*sqr
t(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) -
4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt
(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqr
t(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) +
 2) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log
(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 2*sqrt(6)*(
sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(
3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^
3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
-4*sqrt(3) + 8) + 12*x^2 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x
^3)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8
) + 12*x^2 + 12) - 32)/x^3
```

giac [**A**] time = 0.44, size = 258, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/3/x^3
```

maple [**C**] time = 0.01, size = 46, normalized size = 0.12

$$-\frac{\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^4\ln\left(-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)+x\right)}{4\left(2\,\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^7-\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)^3\right)}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3-1/4*sum(1/(2*_R^7-_R^3)*_R^4*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3\,x^3}-\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

mupad [**B**] time = 0.07, size = 479, normalized size = 1.29

$$-\frac{1}{3x^3}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}-\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12}-\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}+\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2)*8i
)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1
/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^
(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(
3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*
1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/
4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2)
)/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/
4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/
2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/
2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(
1/2)*1i + 1)^(1/4))/12
```

sympy [A] time = 3.17, size = 32, normalized size = 0.09

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-18432t^5 + 4t + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x))) - 1/(3*x**3)
```

3.53 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=280

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2\right.}{a^4\sqrt{b^2-4ac}\,\left(a\right.}$$

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(a^2c^2d-3ab^2cd+2abc^2e-b^3ce+b^4d\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e+4ab^2c^2e-5ab^3cd-b^4ce+b^5d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}+\frac{x\left(a^2d^2+ae(bd-ce)+b^2e^2\right)}{a^3e^3}-\frac{x^2(ad+be)}{2a^2e^2}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{x^3}{3ae}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^5}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{a^2d^2+b^2e^2+ae(bd-ce)}{a^3e^3} - \frac{(ad+be)x}{a^2e^2} + \frac{x^2}{ae} + \frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)} \cdots\right.$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \int \frac{c\left(b^3a \cdots\right.}{\cdots}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^4d - \cdots\right.}{\cdots}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^4d - \cdots\right.}{\cdots}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} + \frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4 \cdots\right.}{a^4\sqrt{b^2-4 \cdots}}$$

Mathematica [A] time = 0.23, size = 283, normalized size = 1.01

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-bde+ce^2\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^4\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}+\frac{x\left(a^2d^2+abde-ace^2+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-bde+ce^2\right)}+\frac{x^3}{3ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(
2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4
*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(
a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a
*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + c/x^2 + b/x)*(d + e*x)), x]
```

fricas [A] time = 94.90, size = 1027, normalized size = 3.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*
e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((
a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a
^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4
 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2
+ 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*
x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^
3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 -
 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 - 6*((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^
5)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)]
```

giac [A] time = 0.38, size = 295, normalized size = 1.05

$$-\frac{d^5\log\left(|xe+d|\right)}{ad^2e^4-bde^5+ce^6}+\frac{\left(b^4d-3\,ab^2cd+a^2c^2d-b^3ce+2\,abc^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^5d^2-a^4bde+a^4ce^2\right)}-\frac{\left(b^5d-5\,ab^3cd+5\,a^2bc^2d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^5d^2-a^4bde+a^4ce^2\right)\sqrt{-b^2+4\,ac}}+\frac{\left(2\,a^2x^3e^2-3\,a^2dx^2e+6\,a^2d^2x-3\,abx^2e^2+6\,abdxe+6\,b^2xe^2-6\,acxe^2\right)e^{(-3)}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^5*log(abs(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2
*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 - a
^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*
a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2
 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*x^3*e^2 - 3*a^2*
d*x^2*e + 6*a^2*d^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x
*e^2)*e^(-3)/a^3
```

maple [B] time = 0.01, size = 662, normalized size = 2.36

$$-\frac{5b\,c^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{2c^3e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{5b^3cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^3}-\frac{4b^2c^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^3}-\frac{b^5d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^4}+\frac{b^4ce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^4}+\frac{c^2d\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^2}-\frac{3b^2cd\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^3}+\frac{b\,c^2e\ln\left(a\,x^2+bx+c\right)}{\left(a\,d^2-deb+c\,e^2\right)a^3}+\frac{b^4d\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^4}-\frac{b^3ce\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^4}-\frac{d^5\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)e^4}+\frac{x^3}{3ae}-\frac{d\,x^2}{2a\,e^2}-\frac{b\,x^2}{2a^2e}+\frac{d^2x}{a\,e^3}+\frac{bdx}{a^2e^2}-\frac{cx}{a^2e}+\frac{b^2x}{a^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/3*x^3/a/e-1/2/a/e^2*x^2*d-1/2/a^2/e*x^2*b+1/a/e^3*d^2*x+1/a^2/e^2*b*d*x-1
/a^2/e*c*x+1/a^3/e*b^2*x+1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*d-
3/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^2*c*d+1/(a*d^2-b*d*e+c*e^2)/a
^3*ln(a*x^2+b*x+c)*b*c^2*e+1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^4*
d-1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^3*c*e-5/(a*d^2-b*d*e+c*e^2)
/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d+2/(a*d^2
-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3
*e+5/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*b^3*c*d-4/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*b^2*c^2*e-1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d+1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-
b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*c*e-1/e^4*d^5/(a*d^2-b*d
*e+c*e^2)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.21, size = 2490, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e^
6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2
- 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 2*a^3*c^3*d^3*e^4 - 4*a^
4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(
b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b
^2 - 4*a*c)^(1/2) + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e
*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^2*c^3*e
^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^
(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 15*a*b^3*c^2*d^2*e^5 + 15*a^2
*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^3*d^2*e^5*
x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(
1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c
)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^3*c*d^3*e^4*x - 5
*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^
2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3
*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d
*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*
e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c*d^2*e^5*x + 9*a^2
*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1
/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2
- 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^5*d*(b^2 - 4
*a*c)^(1/2) - b^6*d + 4*a^3*c^3*d + b^5*c*e - 13*a^2*b^2*c^2*d + 7*a*b^4*c*
d - b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*e + 8*a^2*b*c^3*e - 2*a^2*c^3
*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*d*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^2*e*
(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a
^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 + a^4*b^3*d*e - 4*a^5*b*c*d*e))
- (d^5*log(d + e*x))/(c*e^6 + a*d^2*e^4 - b*d*e^5) - x*((b*d + c*e)/(a^2*e^
2) - (a*d + b*e)^2/(a^3*e^3)) + (log(a^4*b^2*d^7 - 4*a^5*c*d^7 - b^3*c^3*e^
7 + b^6*d^3*e^4 + 6*a^2*c^4*d*e^6 + 3*b^4*c^2*d*e^6 - 3*b^5*c*d^2*e^5 + 2*a
^2*c^4*e^7*x - b^2*c^3*e^7*(b^2 - 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^
(1/2) - 2*a^3*c^3*d^3*e^4 + 4*a^4*c^2*d^5*e^2 + 3*a*b*c^4*e^7 + a^4*b*d^7*(
b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a
*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 8*a^5*c*d^6*e*x + 9*a^2
*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 12*a*b^2*c^3*d*e^6 -
 6*a*b^4*c*d^3*e^4 - a*b^2*c^3*e^7*x + a*b^5*d^3*e^4*x + 2*a^4*b^2*d^6*e*x
+ 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2)
 + 15*a*b^3*c^2*d^2*e^5 - 15*a^2*b*c^3*d^2*e^5 - a^3*b^2*c*d^5*e^2 - a^3*b^
3*d^5*e^2*x - 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) +
 a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2)
- 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(
1/2) - 5*a^2*b^3*c*d^3*e^4*x + 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*
```

```
(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5
*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 12*a^2
*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b
^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^3*c^2*d*e^6
*x - 3*a*b^4*c*d^2*e^5*x - 9*a^2*b*c^3*d*e^6*x + 4*a^4*b*c*d^5*e^2*x + 3*a*
b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/
2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2
 - 4*a*c)^(1/2))*(4*a^3*c^3*d - b^5*d*(b^2 - 4*a*c)^(1/2) - b^6*d + b^5*c*e
 - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d + b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c
^2*e + 8*a^2*b*c^3*e + 2*a^2*c^3*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d*(b^2
 - 4*a*c)^(1/2) - 4*a*b^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*d*(b^2 - 4*
a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2
+ a^4*b^3*d*e - 4*a^5*b*c*d*e)) + x^3/(3*a*e) - (x^2*(a*d + b*e))/(2*a^2*e^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.54 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(-2abcd+ac^2e+b^3d-b^2ce\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{x(ad+be)}{a^2e^2}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tanh^{-1}\left(\ldots\right.}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

Rubi [A] time = 0.40, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2abcd+ac^2e-b^2ce+b^3d\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e-b^3ce+b^4d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{x(ad+be)}{a^2e^2}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{x^2}{2ae}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\int \frac{x^4}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{-ad-be}{a^2e^2}+\frac{x}{ae}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-c\left(b^2d-acd-bce\right)-\left(b^3\right.}{a^2\left(ad^2-e(bd-c\right.}\right.$$

$$=-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-c\left(b^2d-acd-bce\right)-\left(b^3d-2abcd-b^2ce+ac^2e\right)}{c+bx+ax^2}}{a^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\int \frac{b+2}{c+bx}}{2a^3\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\log(c}{2a^3\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{\left(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] time = 0.17, size = 218, normalized size = 1.00

$$\frac{\left(2abcd-ac^2e+b^3(-d)+b^2ce\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)}-\frac{x(ad+be)}{a^2e^2}+\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}+\frac{d^4\log(d+ex)}{e^3\left(ad^2+e(ce-bd)\right)}+\frac{x^2}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3
*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(
a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Lo
g[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + c/x^2 + b/x)*(d + e*x)), x]
```

fricas [A] time = 52.35, size = 798, normalized size = 3.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^2
 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4 -
 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*c)*
log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*
x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*
b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))
/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c -
4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 -
4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4)*x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) -
2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*e^3 + (a
*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d*e^3 - (b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/((a^4*b^2 - 4*a^5
*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)]
```

giac [A] time = 0.37, size = 224, normalized size = 1.03

$$\frac{d^4\log(|xe+d|)}{ad^2e^3-bde^4+ce^5}-\frac{\left(b^3d-2\,abcd-b^2ce+ac^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4d^2-a^3bde+a^3ce^2\right)\sqrt{-b^2+4\,ac}}+\frac{\left(ax^2e-2\,adx-2\,bxe\right)e^{(-2)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^4*log(abs(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c*
d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^
2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 +
 4*a*c)) + 1/2*(a*x^2*e - 2*a*d*x - 2*b*x*e)*e^(-2)/a^2
```

maple [B] time = 0.01, size = 512, normalized size = 2.35

$$\frac{2c^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a}-\frac{4b^2cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{3b\,c^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^2}+\frac{b^4d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^3}-\frac{b^3ce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;a^3}+\frac{bcd\ln\left(a\,x^2+bx+c\right)}{\left(a\,d^2-deb+c\,e^2\right)a^2}-\frac{c^2e\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^2}-\frac{b^3d\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^3}+\frac{b^2ce\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)a^3}+\frac{d^4\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)e^3}+\frac{x^2}{2ae}-\frac{dx}{a\,e^2}-\frac{bx}{a^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/2*x^2/a/e-1/a/e^2*x*d-1/a^2/e*b*x+1/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+
c)*b*c*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*e-1/2/(a*d^2-b*d*e
+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^3*d+1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+
c)*b^2*c*e+2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*c^2*d-4/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+3/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2
)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*e+1/(a*d^2-b*d*e+c*e^2)/a^3/(4*
a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d-1/(a*d^2-b*d*e+c*e
^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*e+1/e^3
*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.24, size = 2051, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*c
^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^4*
c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*c^2
*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) +
2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 -
4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c
)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/
2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^2*c^3*d*
e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)
^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d^4*e^2 + a
^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1
/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)
^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a
*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2
*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*
d^5*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c^2*d*e^5*x + 3*a*b^3*c*d^2*e^4*x - 4
*a^3*b*c*d^4*e^2*x + 3*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*
e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d
*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^2*c^3*e + b^4*c*e + 6*a*b^3*c*d - b^3*c*
e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^5*c*d^2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3
*b^3*d*e - 4*a^4*b*c*d*e)) + (log(2*a*c^4*e^6 - 4*a^4*c*d^6 + a^3*b^2*d^6 -
 b^2*c^3*e^6 + b^5*d^3*e^3 + 3*b^3*c^2*d*e^5 - 3*b^4*c*d^2*e^4 + b^4*d^3*e^
3*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^4 + 4*a^3*c^2*d^4*e^2 + a^3*b*d^6*(
b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) + 2*a^4*d^6*x*(b^2 - 4*a
*c)^(1/2) - 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) - a*b*c^3
*e^6*x - 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*
e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d^3*e^3
 + a*b^4*d^3*e^3*x + 2*a^3*b^2*d^5*e*x - 6*a^2*c^3*d*e^5*x + 3*b^2*c^2*d*e^
5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^2*
d^2*e^4 + 5*a^2*b*c^2*d^3*e^3 - a^2*b^2*c*d^4*e^2 - a^2*b^3*d^4*e^2*x + 2*a
^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^
3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3
*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b*c^
2*d^2*e^4*x - 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*d^5*e*x*(b^2 - 4*a*c)^
(1/2) + 3*a*b^2*c^2*d*e^5*x - 3*a*b^3*c*d^2*e^4*x + 4*a^3*b*c*d^4*e^2*x + 3
*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^(1
/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^5*d + b^4*d*(b^2 - 4*a*c)
^(1/2) - 4*a^2*c^3*e - b^4*c*e - 6*a*b^3*c*d - b^3*c*e*(b^2 - 4*a*c)^(1/2)
+ 8*a^2*b*c^2*d + 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2
*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^
2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3*b^3*d*e - 4*a^4*b*c*d
*e)) + x^2/(2*a*e) - (x*(a*d + b*e))/(a^2*e^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.55 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=176

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}$$

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-3abcd+2ac^2e-b^2ce+b^3d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\int \frac{x^3}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{1}{ae}+\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{a\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$=\frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{c+bx+ax^2}\,dx}{a\left(ad^2-bde+ce^2\right)}$$

$$=\frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-3abcd-b^2ce \ldots\right.}{2a^2\left(ad^2 \ldots\right.}$$

$$=\frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^3d-3abcd- \ldots\right.}{\ldots}$$

$$=\frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2 \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.19, size = 178, normalized size = 1.01

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-bde+ce^2\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^
3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Lo
g[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[x/((a + c/x^2 + b/x)*(d + e*x)), x]
```

fricas [A] time = 16.04, size = 596, normalized size = 3.39

$$\left[-\frac{2\left(a^2b^2-4a^3c\right)d^3\log(ex+d)-\left(\left(b^3-3abc\right)de^2-\left(b^2c-2ac^2\right)e^3\right)\sqrt{b^2-4ac}\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)-2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)x-\left(\left(b^4-5ab^2c+4a^2c^2\right)de^2-\left(b^3c-4abc^2\right)e^3\right)\log\left(ax^2+bx+c\right)}{2\left(\left(a^3b^2-4a^4c\right)d^2e^2-\left(a^2b^3-4a^3bc\right)de^3+\left(a^2b^2c-4a^3c^2\right)e^4\right)},-\frac{2\left(a^2b^2-4a^3c\right)d^3\log(ex+d)-2\left(\left(b^3-3abc\right)de^2-\left(b^2c-2ac^2\right)e^3\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2ax+b)}{b^2-4ac}\right)-2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)x-\left(\left(b^4-5ab^2c+4a^2c^2\right)de^2-\left(b^3c-4abc^2\right)e^3\right)\log\left(ax^2+bx+c\right)}{2\left(\left(a^3b^2-4a^4c\right)d^2e^2-\left(a^2b^3-4a^3bc\right)de^3+\left(a^2b^2c-4a^3c^2\right)e^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (b
^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4*a^
3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b
^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*
x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4), -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - 2*((
b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e
- (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*x + c))/((a
^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)]
```

giac [A] time = 0.40, size = 185, normalized size = 1.05

$$-\frac{d^3 \log\left(|xe + d|\right)}{ad^2e^2 - bde^3 + ce^4} + \frac{xe^{(-1)}}{a} + \frac{\left(b^2d - acd - bce\right)\log\left(ax^2 + bx + c\right)}{2\left(a^3d^2 - a^2bde + a^2ce^2\right)} - \frac{\left(b^3d - 3\,abcd - b^2ce + 2\,ac^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3d^2 - a^2bde + a^2ce^2\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^3*log(abs(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^(-1)/a + 1/2*(b^
2*d - a*c*d - b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2)
 - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [B] time = 0.01, size = 388, normalized size = 2.20

$$\frac{3bcd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac - b^2}\;a} - \frac{2c^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac - b^2}\;a} - \frac{b^3d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac - b^2}\;a^2} + \frac{b^2ce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac - b^2}\;a^2} - \frac{cd\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)a} + \frac{b^2d\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)a^2} - \frac{bce\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)a^2} - \frac{d^3\ln\left(ex + d\right)}{\left(a\,d^2 - deb + c\,e^2\right)e^2} + \frac{x}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] x/a/e-1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*c*d+1/2/(a*d^2-b*d*e+c*e^2)
/a^2*ln(a*x^2+b*x+c)*b^2*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*
e+3/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*b*c*d-2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*e*c^2-1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*l
n(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.34, size = 1367, normalized size = 7.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2*
b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a^2*
b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e
*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2)
+ 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2*d^2*e
^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^
2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 2*a^
2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a
*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2
))*(b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d
 + 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2 - 4*a*c)^(1/2
) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c
^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (log(a^2*b^2*d^5 -
 b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^4*d^3*e^2 + 3*b^
2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2
*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2 - 4*a*c)^(1/2) +
2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^2*c*d^4*e*(b^2 - 4*
a*c)^(1/2) + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^2*d^2*e^3 - 5*a*b^
2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x + 3*a*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 3*b
^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^2*e^3*x + 2*a*b^2*d^3*e^2*x*
(b^2 - 4*a*c)^(1/2) - 3*a^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*e
^4*x - a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^(1
/2) + 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d
^3*e^2*x - 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d + b^3*d*(b^2 - 4*a
*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d + 4*a*b*c^2*e + 2*a*c^2*e*(
b^2 - 4*a*c)^(1/2) - b^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2
*b^3*d*e - 4*a^3*b*c*d*e)) - (d^3*log(d + e*x))/(c*e^4 + a*d^2*e^2 - b*d*e^
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.56 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^2}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-cd-(bd-ce)x}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}+\frac{\int \frac{-cd-(bd-ce)x}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-2acd-bce\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^2d-2acd-bce\right)\mathrm{Subst}\left(\int \frac{}{b^2 \cdots}\right.}{a\left(ad^2-e(bd \cdots\right.}$$

$$= -\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(c+bx \cdots\right.}{2a\left(ad^2-e(bd- \cdots\right.}$$

Mathematica [A] time = 0.12, size = 132, normalized size = 0.89

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ce)\log(x(ax+b)+c)-2ad^2\log(d+ex)\right)+2e\left(2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2ae\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c
]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b
 + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*(d + e*x)), x]
```

fricas [A] time = 5.76, size = 405, normalized size = 2.72

$$\left[\frac{2\left(ab^2-4a^2c\right)d^2\log(ex+d)+\left(bce^2-\left(b^2-2ac\right)de\right)\sqrt{b^2-4ac}\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)-\left(\left(b^3-4abc\right)de-\left(b^2c-4ac^2\right)e^2\right)\log\left(ax^2+bx+c\right)}{2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)}, \frac{2\left(ab^2-4a^2c\right)d^2\log(ex+d)+2\left(bce^2-\left(b^2-2ac\right)de\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2ax+b)}{b^2-4ac}\right)-\left(\left(b^3-4abc\right)de-\left(b^2c-4ac^2\right)e^2\right)\log\left(ax^2+bx+c\right)}{2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)*
sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c
)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*c^2
)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*
c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x
 + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a
*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

giac [A] time = 0.37, size = 149, normalized size = 1.00

$$\frac{d^2\log(|xe+d|)}{ad^2e-bde^2+ce^3}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)}+\frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2d^2-abde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^2*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*x
^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*arc
tan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(-b^
2 + 4*a*c))
```

maple [A] time = 0.01, size = 275, normalized size = 1.85

$$\frac{b^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)\sqrt{4ac-b^2}\,a}-\frac{bce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)\sqrt{4ac-b^2}\,a}-\frac{2cd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)\sqrt{4ac-b^2}}-\frac{bd\ln\left(ax^2+bx+c\right)}{2\left(ad^2-deb+ce^2\right)a}+\frac{ce\ln\left(ax^2+bx+c\right)}{2\left(ad^2-deb+ce^2\right)a}+\frac{d^2\ln(ex+d)}{\left(ad^2-deb+ce^2\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/a*ln
(a*x^2+b*x+c)*c*e-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2))*c*d+1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))/a*b^2*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arc
tan((2*a*x+b)/(4*a*c-b^2)^(1/2))/a*b*c*e+d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.67, size = 966, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^2*log(d + e*x))/(c*e^3 + a*d^2*e - b*d*e^2) - (log(a*b^2*d^4 - 2*c^3*e^4
 - 4*a^2*c*d^4 + b^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*c^2*d^2*e
^2 - 4*b^2*c*d^2*e^2 - b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^
2*d*e^3 - b*c^2*e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 -
4*a*c)^(1/2) + 2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*d^3*e*x + 6*a*c^2*
d*e^3*x - 10*a^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d^
3*e + b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) -
 a*b*d^3*e*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 -
 4*a*c)^(1/2))*(e*((b^2*c)/2 - 2*a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^
3*d)/2 - (b^2*d*(b^2 - 4*a*c)^(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) + 2*a*b*
c*d))/(4*a^3*c*d^2 - a^2*b^2*d^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2
- 4*a^2*b*c*d*e) + (log(2*c^3*e^4 - a*b^2*d^4 + 4*a^2*c*d^4 - b^3*d^3*e + c
^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2 + 4*b^2*c*d^2*e^2 + b^3*d^2
*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^3 + b*c^2*e^4*x + b^2*d^
3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^2*d^4*x*(b^
2 - 4*a*c)^(1/2) - 3*a*b^2*d^3*e*x - 6*a*c^2*d*e^3*x + 10*a^2*c*d^3*e*x - 2
*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^3*e + b^2*d^2*e^2*x*(b^2 - 4*a
*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - a*b*d^3*e*x*(b^2 - 4*a*c)^(1/
2) - a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*d)/2 + e*
(2*a*c^2 - (b^2*c)/2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^2*d*(b^2 - 4*a*c)^
(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d))/(4*a^3*c*d^2 - a^2*b^2*d
^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2 - 4*a^2*b*c*d*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal. Leaf size=124

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx &= \int \frac{x}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{ce+adx}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{\int \frac{ce+adx}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(-bd+2ce)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{(bd-2ce)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{ad^2-e(bd-ce)}\\
&= \frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.86

$$\frac{d\sqrt{4ac-b^2}\,(2\log(d+ex)-\log(x(ax+b)+c))+2(bd-2ce)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c
]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2
) + e*(b*d - c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x*(d + e*x)), x]
```

fricas [A] time = 2.17, size = 305, normalized size = 2.46

$$\left[\frac{\left(b^2-4\,ac\right)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)-\sqrt{b^2-4\,ac}\,(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)},\ \frac{\left(b^2-4\,ac\right)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)+2\sqrt{-b^2+4\,ac}\,(bd-2\,ce)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d)
 - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c -
 sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
- (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*
x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d
 - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 -
4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

giac [A] time = 0.39, size = 127, normalized size = 1.02

$$-\frac{de\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -d*e*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 169, normalized size = 1.36

$$-\frac{bd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{2ce\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}-\frac{d\ln\left(ex+d\right)}{a\,d^2-deb+c\,e^2}+\frac{d\ln\left(a\,x^2+bx+c\right)}{2a\,d^2-2deb+2c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)*d*ln(a*x^2+b*x+c)-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*d+2/(a*d^2-b*d*e+c*e^2)/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c*e-d/(a*d^2-b*d*e+c*e^2)*l
n(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.41, size = 801, normalized size = 6.46

$$\frac{\ln\left(a\,e\,x-\frac{\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(d\,a^2\,e+b\,a\,e^2\right)+\frac{\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\,e^3+2\,a\,b^2\,e^3\right)+a\,b\,c\,e^3+a\,b^2\,d\,e^2+a^2\,b\,d^2\,e-8\,a^2\,c\,d\,e^2\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,c\,e^2+a\,b\,d\,e\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}\right)\,\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}-\frac{\ln\left(\frac{\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(d\,a^2\,e+b\,a\,e^2\right)-\frac{\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\,e^3+2\,a\,b^2\,e^3\right)+a\,b\,c\,e^3+a\,b^2\,d\,e^2+a^2\,b\,d^2\,e-8\,a^2\,c\,d\,e^2\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,c\,e^2+a\,b\,d\,e\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,e\,x\right)\,\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}-\frac{d\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(a*e*x - ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - c*e*(b^2 - 4
*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) + ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*
```

```
c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*a^3
*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2*c*d
*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b
*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*
c*e^2 - b^3*d*e + 4*a*b*c*d*e))*(d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2
/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^
2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(((d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2)
)/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) - ((d*(2*a*c
 + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^
2*e^3 - 6*a^2*c*e^3 + 2*a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^
2 + a^2*b*d^2*e - 8*a^2*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 +
b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2
*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*e*x)*(d*(2*a*
c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d
^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (d*lo
g(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal. Leaf size=123

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1569, 705, 31, 634, 618, 206, 628}

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx &= \int \frac{1}{(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \frac{e^2\int \frac{1}{d+ex}\,dx}{ad^2-bde+ce^2}+\frac{\int \frac{ad-be-aex}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)} \\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(2ad-be)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)} \\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(2ad-be)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{ad^2-e(bd-ce)} \\
&= -\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.85

$$
\frac{e\sqrt{4ac-b^2}\left(\log(x(ax+b)+c)-2\log(d+ex)\right)+(2be-4ad)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^2*(d + e*x)), x]
```

fricas [A] time = 2.05, size = 305, normalized size = 2.48

$$
\left[-\frac{\left(b^2-4\,ac\right)e\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)e\log\left(ex+d\right)+\sqrt{b^2-4\,ac}\,(2\,ad-be)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)},\ -\frac{\left(b^2-4\,ac\right)e\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)e\log\left(ex+d\right)+2\sqrt{-b^2+4\,ac}\,(2\,ad-be)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d
) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2
- 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

giac [A] time = 0.34, size = 126, normalized size = 1.02

$$-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*e*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*log(abs(x*e + d))
/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

maple [A] time = 0.01, size = 168, normalized size = 1.37

$$\frac{2ad\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}-\frac{be\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{e\ln\left(ex+d\right)}{a\,d^2-deb+c\,e^2}-\frac{e\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x)
```

```
[Out] -1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*
c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*e+e*ln(e*x+d)/(a*d^2-b*d
*e+c*e^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.82, size = 521, normalized size = 4.24

$$\frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e+a\,d\,\sqrt{b^2-4\,a\,c}-\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\,d\,e+2\,x\,b^2\,e^2+c\,b\,e^2\right)}{\left(4\,a\,c-b^2\right)\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}\right)\,\left(e\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-a\,d\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}-\frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e-a\,d\,\sqrt{b^2-4\,a\,c}+\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\,d\,e+2\,x\,b^2\,e^2+c\,b\,e^2\right)}{\left(4\,a\,c-b^2\right)\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}\right)\,\left(e\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-a\,d\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+\frac{e\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e + a*d*(b^2
 - 4*a*c)^(1/2) - (b*e*(b^2 - 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x +
```

```
 a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*
a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2
- b^2/2) - a*d*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2
 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*
e - (a*e*((b^2*e)/2 - 2*a*c*e - a*d*(b^2 - 4*a*c)^(1/2) + (b*e*(b^2 - 4*a*c
)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*
c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
)*(e*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - a*d*(b^2 - 4*a*c)^(1/2))
)/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*
e) + (e*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal. Leaf size=158

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

Rubi [A] time = 0.27, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx=\int \frac{1}{x(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{1}{cdx}+\frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{b^2e-a(bd+ce)-a(ad-be)x}{c\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right) d$$

$$=\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{b^2e-a(bd+ce)-a(ad-be)x}{c+bx+ax^2}\,dx}{c\left(ad^2-bde+ce^2\right)}$$

$$=\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\left(-abd+b^2e-2ace\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\int}{2c\left(ad^2-e\right.}$$

$$=\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)}-\frac{\left(-abd+b^2e-2ac\right.}{}$$

$$=\frac{\left(abd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-bde+ce^2\right)}+\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)}{2c\left(a\right.}$$

Mathematica [A] time = 0.19, size = 152, normalized size = 0.96

$$-\frac{\sqrt{4ac-b^2}\left(-2\log(x)\left(ad^2+e(ce-bd)\right)+d(ad-be)\log(x(ax+b)+c)+2ce^2\log(d+ex)\right)+2d\left(abd+2ace+b^2(-e)\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2cd\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
+ Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d
+ e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a*d^
2 + e*(-(b*d) + c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^3*(d + e*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 164, normalized size = 1.04

$$-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2\left(acd^2-bcde+c^2e^2\right)}-\frac{e^3\log\left(|xe+d|\right)}{ad^3e-bd^2e^2+cde^3}-\frac{\left(abd-b^2e+2\,ace\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(acd^2-bcde+c^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(|x|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*d - b*e)*log(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - e^3*l
og(abs(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d - b^2*e + 2*a*c*e
)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*sqr
t(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

maple [A] time = 0.01, size = 285, normalized size = 1.80

$$-\frac{abd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{2ae\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}}+\frac{b^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{ad\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)c}+\frac{be\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)c}-\frac{e^2\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)d}+\frac{\ln(x)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)/c*a*ln(a*x^2+b*x+c)*d+1/2/(a*d^2-b*d*e+c*e^2)/c*ln
(a*x^2+b*x+c)*b*e-1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a*b*d-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2))*a*e+1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e+ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b
*d*e+c*e^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.40, size = 2399, normalized size = 15.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(b^3*c^3*e^5 - 6*a^4*c^2*d^5 + 2*a^3*b^2*c*d^5 + 8*a^2*c^4*d*e^4 - b^4*
c^2*d*e^4 - 2*b^5*c*d^2*e^3 + 2*a^3*b^3*d^5*x + 8*a^2*c^4*e^5*x + b^4*c^2*e
^5*x - 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*d^3*e
^2 - 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*e^3*(b
^2 - 4*a*c)^(1/2) - 7*a^4*b*c*d^5*x - b^5*c*d*e^4*x - 27*a^2*b^2*c^2*d^3*e^
2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(1/2) +
 2*a*b^2*c^3*d*e^4 + 6*a*b^4*c*d^3*e^2 - 6*a^2*b^3*c*d^4*e + 21*a^3*b*c^2*d
```

```
^4*e - 6*a*b^2*c^3*e^5*x + 6*a*b^5*d^3*e^2*x - 6*a^2*b^4*d^4*e*x - 14*a^4*c
^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2 - 4*a
*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^2 - 4*
a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2 - 4*a
*c)^(1/2) + 13*a*b^3*c^2*d^2*e^3 - 21*a^2*b*c^3*d^2*e^3 + 10*a^3*c^3*d^2*e^
3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*(b^2 -
 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*b^3*c*d^3*e^
2*x + 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 13
*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4*a*c)^
(1/2) - 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) - 4*
a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 5*a
*b^3*c^2*d*e^4*x + 14*a*b^4*c*d^2*e^3*x - 4*a^2*b*c^3*d*e^4*x + 26*a^3*b^2*
c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^4*x*(b
^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2
*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2)
)*(d*((a*b^2)/2 - 2*a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/2) - (b^3*e)/2 - (b^2
*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*e))/(4*a*c^
3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b*c^2*d
*e) - (log(6*a^4*c^2*d^5 - b^3*c^3*e^5 - 2*a^3*b^2*c*d^5 - 8*a^2*c^4*d*e^4
+ b^4*c^2*d*e^4 + 2*b^5*c*d^2*e^3 - 2*a^3*b^3*d^5*x - 8*a^2*c^4*e^5*x - b^4
*c^2*e^5*x + 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3
*d^3*e^2 + 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*
e^3*(b^2 - 4*a*c)^(1/2) + 7*a^4*b*c*d^5*x + b^5*c*d*e^4*x + 27*a^2*b^2*c^2*
d^3*e^2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(
1/2) - 2*a*b^2*c^3*d*e^4 - 6*a*b^4*c*d^3*e^2 + 6*a^2*b^3*c*d^4*e - 21*a^3*b
*c^2*d^4*e + 6*a*b^2*c^3*e^5*x - 6*a*b^5*d^3*e^2*x + 6*a^2*b^4*d^4*e*x + 14
*a^4*c^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2
 - 4*a*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^
2 - 4*a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2
 - 4*a*c)^(1/2) - 13*a*b^3*c^2*d^2*e^3 + 21*a^2*b*c^3*d^2*e^3 - 10*a^3*c^3*
d^2*e^3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2
- 4*a*c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*
(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*b^3*c*
d^3*e^2*x - 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2
) - 13*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4
*a*c)^(1/2) + 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) - 4*a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2)
 - 5*a*b^3*c^2*d*e^4*x - 14*a*b^4*c*d^2*e^3*x + 4*a^2*b*c^3*d*e^4*x - 26*a^
3*b^2*c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^
4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2
*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)
^(1/2))*((b^3*e)/2 + d*(2*a^2*c - (a*b^2)/2 + (a*b*(b^2 - 4*a*c)^(1/2))/2)
- (b^2*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*e))/(
4*a*c^3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b
*c^2*d*e) - (e^2*log(d + e*x))/(a*d^3 - b*d^2*e + c*d*e^2) + log(x)/(c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
[Out] Timed out
```

3.60 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal. Leaf size=193

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}$$

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx=\int \frac{1}{x^2(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{1}{cdx^2}+\frac{-bd-ce}{c^2d^2x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-a^2cd-b^3e+ab(bd+2c$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-a^2cd-b^3e+ab(bd+2ce)+a\left(abd-b^2e+\right.}{c+bx+ax^2}}{c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\int \frac{b+2ax}{c+bx+ax^2}\,d}{2c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\log\left(c+bx\right.}{2c^2\left(ad^2-e(bd-ce)\right)}$$

$$=-\frac{1}{cdx}+\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{}{d^2(}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 1.01

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\left(ad^2+e(ce-bd)\right)}+\frac{e^3\log(d+ex)}{ad^4+d^2e(ce-bd)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) -
((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d)
+ c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^4*(d + e*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 210, normalized size = 1.09

$$\frac{\left(abd - b^2e + ace\right)\log\left(ax^2 + bx + c\right)}{2\left(ac^2d^2 - bc^2de + c^3e^2\right)} + \frac{e^4\log\left(|xe + d|\right)}{ad^4e - bd^3e^2 + cd^2e^3} + \frac{\left(ab^2d - 2\,a^2cd - b^3e + 3\,abce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ac^2d^2 - bc^2de + c^3e^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{(bd + ce)\log\left(|x|\right)}{c^2d^2} - \frac{1}{cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(a*b*d - b^2*e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c
^3*e^2) + e^4*log(abs(x*e + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + (a*b^2*
d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(
(a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c)) - (b*d + c*e)*log(abs
(x))/(c^2*d^2) - 1/(c*d*x)
```

maple [B] time = 0.01, size = 412, normalized size = 2.13

$$-\frac{2a^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}+\frac{a\,b^2d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c^2}+\frac{3abe\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c}-\frac{b^3e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)\sqrt{4ac-b^2}\;c^2}+\frac{abd\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)c^2}+\frac{ae\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)c}-\frac{b^2e\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)c^2}+\frac{e^3\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)d^2}-\frac{b\ln(x)}{c^2d}-\frac{e\ln(x)}{c\,d^2}-\frac{1}{cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/c
*a*ln(a*x^2+b*x+c)*e-1/2/(a*d^2-b*d*e+c*e^2)/c^2*ln(a*x^2+b*x+c)*b^2*e-2/(a
*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a
^2*d+1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^
2)^(1/2))*a*b^2*d+3/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a*b*e-1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e-1/c/d/x-1/c^2/d*ln(x)*b-1/c/d^2*ln(x)
*e+e^3/(a*d^2-b*d*e+c*e^2)/d^2*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 20.39, size = 2388, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (e^3*log(d + e*x))/(a*d^4 + c*d^2*e^2 - b*d^3*e) + (log((a^4*e^4*x)/(c^2*d^
2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^
2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
 a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e
+ b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*
```

```
d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*
a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c
*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2
) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*
c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(
4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c
^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x -
8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b
*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
 - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^
4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a
*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*
a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
 + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e +
b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c
*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*
e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2
- a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) + (log((a^4*e^4*x)/(c^2*d^2
) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^2
*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e +
 b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*d
) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*a
^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c*
d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2)
 + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c
)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(4
*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^
2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8
*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*
c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
- 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
+ (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e - b
^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*
e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e
*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*
e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b
^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 -
 a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - 1/(c*d*x) - (log(x)*(b*d +
 c*e))/(c^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal. Leaf size=252

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log}{}$$

Rubi [A] time = 0.43, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)\left(-c\left(ad^2-ce^2\right)+b^2d^2+bcde\right)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e)
- a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*e^
2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) +
((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2
- e*(b*d - c*e)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx &= \int \frac{1}{x^3(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{cdx^3}+\frac{-bd-ce}{c^2d^2x^2}+\frac{b^2d^2+bcde-c\left(ad^2-ce^2\right)}{c^3d^3x}+\frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+\right. \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}-\frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 252, normalized size = 1.00

$$
\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}+\frac{\log(x)\left(c\left(ce^2-ad^2\right)+b^2d^2+bcde\right)}{c^3d^3}-\frac{e^4\log(d+ex)}{ad^5+d^3e(ce-bd)}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[
-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^
2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d)
+ c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c
^3*(a*d^2 + e*(-(b*d) + c*e)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^5*(d + e*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 279, normalized size = 1.11

$$-\frac{\left(ab^2d - a^2cd - b^3e + 2\,abce\right)\log\left(ax^2 + bx + c\right)}{2\left(ac^3d^2 - bc^3de + c^4e^2\right)} - \frac{e^5\log\left(|xe + d|\right)}{ad^5e - bd^4e^2 + cd^3e^3} - \frac{\left(ab^3d - 3\,a^2bcd - b^4e + 4\,ab^2ce - 2\,a^2c^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ac^3d^2 - bc^3de + c^4e^2\right)\sqrt{-b^2 + 4\,ac}} + \frac{\left(b^2d^2 - acd^2 + bcde + c^2e^2\right)\log\left(|x|\right)}{c^3d^3} - \frac{c^2d^2 - 2\left(bcd^2 + c^2de\right)x}{2\,c^3d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^
2 - b*c^3*d*e + c^4*e^2) - e^5*log(abs(x*e + d))/(a*d^5*e - b*d^4*e^2 + c*d
^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arcta
n((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-
b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c^3*d^
3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

maple [B] time = 0.01, size = 562, normalized size = 2.23

$$\frac{3a^2bd\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac-b^2}\;c^2} + \frac{2a^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac-b^2}\;c} - \frac{a\,b^3d\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac-b^2}\;c^3} - \frac{4a\,b^2e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac-b^2}\;c^2} + \frac{b^4e\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)\sqrt{4ac-b^2}\;c^3} + \frac{a^2d\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)c^2} - \frac{a\,b^2d\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)c^3} - \frac{abe\ln\left(a\,x^2 + bx + c\right)}{\left(a\,d^2 - deb + c\,e^2\right)c^2} + \frac{b^3e\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)c^3} - \frac{e^4\ln\left(ex + d\right)}{\left(a\,d^2 - deb + c\,e^2\right)d^3} - \frac{a\ln(x)}{c^2d} + \frac{b^2\ln(x)}{c^3d} + \frac{be\ln(x)}{c^2d^2} + \frac{e^2\ln(x)}{c\,d^3} + \frac{b}{c^2dx} + \frac{e}{c\,d^2x} - \frac{1}{2cd\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)/c^2*a^2*ln(a*x^2+b*x+c)*d-1/2/(a*d^2-b*d*e+c*e^2)/c
^3*a*ln(a*x^2+b*x+c)*b^2*d-1/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*e+
1/2/(a*d^2-b*d*e+c*e^2)/c^3*ln(a*x^2+b*x+c)*b^3*e+3/(a*d^2-b*d*e+c*e^2)/c^2
/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d+2/(a*d^2-b*d
*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/(
a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)
)*a*b^3*d-4/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a
*c-b^2)^(1/2))*a*b^2*e+1/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/c/d/x^2+1/c^2/d/x*b+1/c/d^2/x*e-1/c^2
/d*ln(x)*a+1/c^3/d*ln(x)*b^2+1/c^2/d^2*ln(x)*b*e+1/c/d^3*ln(x)*e^2-e^4/(a*d
^2-b*d*e+c*e^2)/d^3*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 26.16, size = 3530, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*e
*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e +
```

```
b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^
4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2
*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*
a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4
*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*
a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e
^3 + 10*a^2*b^2*c*d^3*e^2))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^(1/2) - b
^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) -
5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c
*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e
+ b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4
*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x -
2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6
*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2
*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c
)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*
c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^
3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(a^3*b^3*d^6 + b^3*c^3*e^6
+ b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*e + 2*b^4*c^2*d*e^5 + 2*b^5*c
*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3*a*b*c^4*e^6 + 9*a^2*b^2*c^2*
d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^3 - 9*a*b^3*c^2*d^2*e^4 + 7*a
^2*b*c^3*d^2*e^4))/(c^4*d^4) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4*c^
2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^4*e
^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b^4*
c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^
5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) - 5
*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b
^2)*(a*d^2 + c*e^2 - b*d*e)) - (a^5*e^5*x)/(c^3*d^3))*(b^4*e*(b^2 - 4*a*c)^
(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)
^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a
*c^5*e^2 + 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*
b*c^4*d*e)) - (e^4*log(d + e*x))/(a*d^5 + c*d^3*e^2 - b*d^4*e) - (log((((a*
e*(a^3*b^3*d^6 + b^3*c^3*e^6 + b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*
e + 2*b^4*c^2*d*e^5 + 2*b^5*c*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3
*a*b*c^4*e^6 + 9*a^2*b^2*c^2*d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^
3 - 9*a*b^3*c^2*d^2*e^4 + 7*a^2*b*c^3*d^2*e^4))/(c^4*d^4) - (((a*e*(a^2*b^3
*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e + b^3*c^2*
d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^4*d^4*e -
 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2*c^2*d^2*
e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*a^4*c*d^5
 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4*e + 8*a^
2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*a^3*b*c*d
^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e^3 + 10*a
^2*b^2*c*d^3*e^2))/(c^2*d^2) + (a*e*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*
a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^2
*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2
*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*
x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c
*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*
d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/
(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4
```

```
*c^2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^
4*e^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b
^4*c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b
*c*d*e))/(c^4*d^4) - (a^5*e^5*x)/(c^3*d^3))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1
/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5
*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^2 +
 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)
) - (1/(2*c*d) - (x*(b*d + c*e))/(c^2*d^2))/x^2 + (log(x)*(c^2*e^2 - d^2*(a
*c - b^2) + b*c*d*e))/(c^3*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=343

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}+\frac{\left(-4a^2c^3de-b\right.}{}$$

Rubi [A] time = 0.91, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-4a^2c^3de+8ab^2c^2de-b^3c\left(5ad^2-ce^2\right)+abc^2\left(5ad^2-3ce^2\right)-2b^4cde+b^5d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\int \frac{x^5}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$=\int\left(\frac{-2ad-be}{a^2e^3}+\frac{x}{ae^2}+\frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)^2}+\frac{d^4\left(3ad^2-e(4bd-\right.}{e^3\left(ad^2-e(bd-ce)\right)^2}\right.$$

$$=-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right)}{e^4\left(ad^2-e(bd-\right.}$$

$$=-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right)}{e^4\left(ad^2-e(bd-\right.}$$

$$=-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{d^4\left(3ad^2-e(4bd-5ce\right)}{e^4\left(ad^2-e(bd-\right.}$$

$$=-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(b^5d^2-2b^4cde+8ab^2c\right.}{}$$

Mathematica [A] time = 0.36, size = 338, normalized size = 0.99

$$\frac{\left(b^2c\left(ce^2-3ad^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)^2}-\frac{x(2ad+be)}{a^2e^3}-\frac{\left(-4a^2c^3de+b^3c\left(ce^2-5ad^2\right)+8ab^2c^2de+abc^2\left(5ad^2-3ce^2\right)+b^5d^2-2b^4cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{d^5}{e^4(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{\log(d+ex)\left(3ad^6+d^4e(5ce-4bd)\right)}{e^4\left(ad^2+e(ce-bd)\right)^2}+\frac{x^2}{2ae^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^
3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b +
 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c
*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e
*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^
2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + c/x^2 + b/x)*(d + e*x)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.42, size = 565, normalized size = 1.65

$$\frac{d^5e^4}{(ad^2e^8-bde^9+ce^{10})(xe+d)}+\frac{\left(b^5d^2e^2-5\,ab^3cd^2e^2+5\,a^2bc^2d^2e^2-2\,b^4cde^3+8\,ab^2c^2de^3-4\,a^2c^3de^3+b^3c^2e^4-3\,abc^3e^4\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4\right)\sqrt{-b^2+4\,ac}}+\frac{\left(a^2-\frac{2\left(3\,a^2de+abe^2\right)e^{(-1)}}{xe+d}\right)(xe+d)^2e^{(-4)}}{2\,a^3}+\frac{\left(b^4d^2-3\,ab^2cd^2+a^2c^2d^2-2\,b^3cde+4\,abc^2de+b^2c^2e^2-ac^3e^2\right)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^2}{(xe+d)^2}-\frac{be}{xe+d}+\frac{bde}{(xe+d)^2}-\frac{ce^2}{(xe+d)^2}\right)}{2\left(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4\right)}-\frac{\left(3\,a^2d^2+2\,abde+b^2e^2-ace^2\right)e^{(-4)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(x*e + d)) + (b^5*d^2*e^2 - 5*a*b^3
*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 - 4*a^
2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*
a^3*b*c*d*e^3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2 - 2*(3*a^2*d*e
+ a*b*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-4)/a^3 + 1/2*(b^4*d^2 - 3*a*b^
2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*e
^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x
*e + d)^2 - c*e^2/(x*e + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 +
 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) - (3*a^2*d^2 + 2*a*b*d*e
+ b^2*e^2 - a*c*e^2)*e^(-4)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^3
```

maple [B] time = 0.01, size = 943, normalized size = 2.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] 1/2*x^2/a/e^2-2/a/e^3*x*d-1/a^2/e^2*b*x+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^
2+b*x+c)*c^2*d^2-3/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*c*d^2+2/
(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)
^2/a^2*ln(a*x^2+b*x+c)*c^3*e^2+1/2/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c
)*b^4*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^3*c*d*e+1/2/(a*d^2-
b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^2*c^2*e^2-5/(a*d^2-b*d*e+c*e^2)^2/a/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2+4/(a*d^2-b*d*
e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*d*e+
5/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2))*b^3*c*d^2-8/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*e^2-1/(a*d^2-b*d*e+c*e^2)^
2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d^2+2/(a*d^
2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*
b^4*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4
*a*c-b^2)^(1/2))*b^3*c^2*e^2+3/e^4*d^6/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-4/
e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b+5/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*l
n(e*x+d)*c+1/e^4*d^5/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.04, size = 3503, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4 +
 b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (log(12*a^5*c
*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b^3*
d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^(1/2)
 + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4*c^2*
d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*e^8*(b^2 - 4*a*c)^(1/2) -
 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7*e - 4*
a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) + a*b*c^4*e^8*x + 24*a^5*c*d^7*e*
x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^(1/2) + 12*a^4*c*d
^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^4*c*d^4*
e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x + 4*a^3*b^2*d
^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^(1/2) - 4*b^4*c*d^
3*e^5*(b^2 - 4*a*c)^(1/2) - 24*a*b^2*c^3*d^2*e^6 + 20*a*b^3*c^2*d^3*e^5 - 2
0*a^2*b*c^3*d^3*e^5 - 4*a^2*b^3*c*d^5*e^3 + 16*a^3*b*c^2*d^5*e^3 - 2*a^3*b^
2*c*d^6*e^2 - 4*a^2*b^4*d^5*e^3*x + 11*a^3*b^3*d^6*e^2*x - 8*a^3*c^3*d^3*e^
5*x + 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b^2 - 4*a*c)^(1/2) - 4*a*b
^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*e^2*(b^2 - 4*a*c)^(1/2) +
 a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6*e^2*x*(b^2 - 4*a*c)^(1/2
) + 6*a*b^3*c^2*d^2*e^6*x - 18*a^2*b*c^3*d^2*e^6*x - 15*a^3*b*c^2*d^4*e^4*x
 + 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b^2 - 4*a*c)^(1/2) - 2*a^2
*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a^3*b^2*d^6*e^2*x*(b^2 - 4
*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 11*a^3*c^2*d^4*e^4*
x*(b^2 - 4*a*c)^(1/2) + 16*a^2*b^2*c^2*d^3*e^5*x + 14*a^4*b*d^7*e*x*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c^3*d*e^7*x - 4*a*b^4*c*d^3*e^5*x - 44*a^4*b*c*d^6*e
^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^3*e^5*x*(b^2 - 4
*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^6
*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*
b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 + b^5*d^2*(b^2 - 4*a*c)^(1/2)
 - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5*a*b^2*c^3*e^2 + b^3*c^2*
e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2*c^2*d^2 - 7*a*b^4*c*d^2
+ 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e - 5*a*b^3*c*d^2*(b^2 - 4*a*c)^(1/2) -
 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d*e*(b^2 - 4*a*c)^(1/2) + 5*
a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^4*c*d*e*(b^2 - 4*a*c)^(1/2) + 8*a*b
^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 - a^5*b^2*d^4 + 4*a^4*c^3*
e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*d^2*e^2 + 8*a^5*c^2*d^2*e
^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b*c^2*d*e^3 + 2*a^4*b^2*c*
d^2*e^2)) - (log(2*a*c^5*e^8 - 12*a^5*c*d^8 + 3*a^4*b^2*d^8 - b^2*c^4*e^8 -
 b^6*d^4*e^4 - 4*a^3*b^3*d^7*e + 4*b^3*c^3*d*e^7 + 4*b^5*c*d^3*e^5 + b^5*d^
4*e^4*(b^2 - 4*a*c)^(1/2) - 12*a^2*c^4*d^2*e^6 + 22*a^3*c^3*d^4*e^4 - 8*a^4
*c^2*d^6*e^2 - 6*b^4*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*
e^8*(b^2 - 4*a*c)^(1/2) - 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^4*d*e^
7 + 16*a^4*b*c*d^7*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^
```

```
4*e^8*x - 24*a^5*c*d^7*e*x - 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 -
4*a*c)^(1/2) + 12*a^4*c*d^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*c*d^4*e^4 - a*b^5*d^4*e^4*x + 6*a^4*b^2*d^7*e*x - 8*a^2*
c^4*d*e^7*x + 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 -
4*a*c)^(1/2) - 4*b^4*c*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 24*a*b^2*c^3*d^2*e^6 -
 20*a*b^3*c^2*d^3*e^5 + 20*a^2*b*c^3*d^3*e^5 + 4*a^2*b^3*c*d^5*e^3 - 16*a^3
*b*c^2*d^5*e^3 + 2*a^3*b^2*c*d^6*e^2 + 4*a^2*b^4*d^5*e^3*x - 11*a^3*b^3*d^6
*e^2*x + 8*a^3*c^3*d^3*e^5*x - 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b
^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*
e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6
*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*d^2*e^6*x + 18*a^2*b*c^3*d^2*e^6*x
 + 15*a^3*b*c^2*d^4*e^4*x - 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b
^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a
^3*b^2*d^6*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1
/2) + 11*a^3*c^2*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 16*a^2*b^2*c^2*d^3*e^5*x +
 14*a^4*b*d^7*e*x*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^3*d*e^7*x + 4*a*b^4*c*d^3
*e^5*x + 44*a^4*b*c*d^6*e^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a
*b^3*c*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1
/2) + 6*a*b^2*c^2*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 - b^
5*d^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5
*a*b^2*c^3*e^2 - b^3*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2
*c^2*d^2 - 7*a*b^4*c*d^2 + 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e + 5*a*b^3*c*
d^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e
*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e*(b
^2 - 4*a*c)^(1/2) - 8*a*b^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 -
 a^5*b^2*d^4 + 4*a^4*c^3*e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*
d^2*e^2 + 8*a^5*c^2*d^2*e^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b
*c^2*d*e^3 + 2*a^4*b^2*c*d^2*e^2)) + x^2/(2*a*e^2) - (x*(b*e^2 + 2*a*d*e))/
(a^2*e^4) + (a^2*d^5)/(e*(a^2*d*e^3 + a^2*e^4*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.63 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=274

$$-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-\cdots\right.}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Rubi [A] time = 0.56, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)\left(2ad^2-e(3bd-4ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{x}{ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= \int \frac{x^4}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{ae^2}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{d^3\left(-2ad^2+e(3bd-4ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2(d+ex)}+\frac{-c\left(b^2\right.}{} \right. \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\int \frac{-c}{}}{} \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2}-\frac{((bd}{} \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-}{} \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2\left(ad^2-ce^2\right)\right.}{a^2\sqrt{b^2-4ac}\left(ad^2-\right.}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 269, normalized size = 0.98

$$\frac{(bd-ce)\left(2acd+b^2(-d)+bce\right)\log(x(ax+b)+c)}{2a^2\left(ad^2+e(ce-bd)\right)^2}+\frac{\left(b^2c\left(ce^2-4ad^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2+e(ce-bd)\right)}-\frac{\log(d+ex)\left(2ad^5+d^3e(4ce-3bd)\right)}{e^3\left(ad^2+e(ce-bd)\right)^2}+\frac{x}{ae^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*
b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e
^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2
 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])
/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*
c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + c/x^2 + b/x)*(d + e*x)^2), x]
```

fricas [B] time = 158.65, size = 2139, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b
^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4
*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c
- 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)
*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e
^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^
3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*c^2
- 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b^4*c
 - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b^5 -
 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e
^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2 - 4
*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)*d^4
*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + 4*(
a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d^5*e
^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*
d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*d
*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 + (a^
2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^
7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x), -1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*
(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b
^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^
2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4
*a^2*c^3)*e^6)*x^2 + 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c -
3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*
c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sq
rt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*
((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a
^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*
c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)
*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2
 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)
*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 +
 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d
^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c
^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 +
 (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d*e^7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x)]
```

giac [A] time = 0.40, size = 476, normalized size = 1.74

$$-\frac{d^{4} e^{3}}{\left(a d^{2} e^{6} - b d e^{7} + c e^{8}\right)(x e + d)} - \frac{\left(b^{4} d^{2} e^{2} - 4\, a b^{2} c d^{2} e^{2} + 2\, a^{2} c^{2} d^{2} e^{2} - 2\, b^{3} c d e^{3} + 6\, a b c^{2} d e^{3} + b^{2} c^{2} e^{4} - 2\, a c^{3} e^{4}\right) \arctan\left(-\frac{\left(2\, a d - \frac{2\, a d^{2}}{x e + d} - b e + \frac{2\, b d e}{x e + d} - \frac{2\, c e^{2}}{x e + d}\right) e^{(-1)}}{\sqrt{-b^{2} + 4\, a c}}\right) e^{(-2)}}{\left(a^{4} d^{4} - 2\, a^{3} b d^{3} e + a^{2} b^{2} d^{2} e^{2} + 2\, a^{3} c d^{2} e^{2} - 2\, a^{2} b c d e^{3} + a^{2} c^{2} e^{4}\right) \sqrt{-b^{2} + 4\, a c}} + \frac{(x e + d) e^{(-3)}}{a} - \frac{\left(b^{3} d^{2} - 2\, a b c d^{2} - 2\, b^{2} c d e + 2\, a c^{2} d e + b c^{2} e^{2}\right) \log\left(-a + \frac{2\, a d}{x e + d} - \frac{a d^{2}}{(x e + d)^{2}} - \frac{b e}{x e + d} + \frac{b d e}{(x e + d)^{2}} - \frac{c e^{2}}{(x e + d)^{2}}\right)}{2 \left(a^{4} d^{4} - 2\, a^{3} b d^{3} e + a^{2} b^{2} d^{2} e^{2} + 2\, a^{3} c d^{2} e^{2} - 2\, a^{2} b c d e^{3} + a^{2} c^{2} e^{4}\right)} + \frac{(2\, a d + b e) e^{(-3)} \log\left(\frac{|x e + d| e^{(-1)}}{(x e + d)^{2}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(x*e + d)) - (b^4*d^2*e^2 - 4*a*b^2
*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*
e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e
+ d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^4*d^4 - 2*a
^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*
e^4)*sqrt(-b^2 + 4*a*c)) + (x*e + d)*e^(-3)/a - 1/2*(b^3*d^2 - 2*a*b*c*d^2
- 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(
```

```
x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^4*d^
4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a
^2*c^2*e^4) + (2*a*d + b*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^2
```

maple [B] time = 0.01, size = 765, normalized size = 2.79

$$-\frac{4b^2cd^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a}+\frac{6bc^2de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a}-\frac{2c^3e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a}+\frac{b^4d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a^2}-\frac{2b^3cde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a^2}+\frac{b^2c^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}\,a^2}+\frac{2c^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-deb+ce^2\right)^2\sqrt{4ac-b^2}}-\frac{2ad^5\ln\left(ex+d\right)}{\left(ad^2-deb+ce^2\right)^2e^3}+\frac{bcd^2\ln\left(ax^2+bx+c\right)}{\left(ad^2-deb+ce^2\right)^2a}-\frac{c^2de\ln\left(ax^2+bx+c\right)}{\left(ad^2-deb+ce^2\right)^2a}-\frac{b^3d^2\ln\left(ax^2+bx+c\right)}{2\left(ad^2-deb+ce^2\right)^2a^2}+\frac{b^2cde\ln\left(ax^2+bx+c\right)}{\left(ad^2-deb+ce^2\right)^2a^2}-\frac{bc^2e^2\ln\left(ax^2+bx+c\right)}{2\left(ad^2-deb+ce^2\right)^2a^2}+\frac{3bd^4\ln\left(ex+d\right)}{\left(ad^2-deb+ce^2\right)^2e^2}-\frac{4cd^3\ln\left(ex+d\right)}{\left(ad^2-deb+ce^2\right)^2e}-\frac{d^4}{\left(ad^2-deb+ce^2\right)\left(ex+d\right)e^3}+\frac{x}{ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d^2-1/(a*d^2-b*d*e+c*
e^2)^2/a*ln(a*x^2+b*x+c)*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x
+c)*b^3*d^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*c*d*e-1/2/(a*d^
2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2-4/(a*d^2-b*d*e+c*
e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d^2+6/
(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*b*c^2*d*e-2/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2))*c^3*e^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d^2-2/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e+1/(a*d^2-b*d*e+c*
e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*e^
2-1/e^3*d^4/(a*d^2-b*d*e+c*e^2)/(e*x+d)-2/e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(
e*x+d)*a+3/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-4/e*d^3/(a*d^2-b*d*e+c
*e^2)^2*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.00, size = 2495, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e^2) - (log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 + a
^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) + (
log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^(1/2) - 2*a^3*b^2*d^7 +
 b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^4*d^
4*e^3*(b^2 - 4*a*c)^(1/2) - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6*b^3*
c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/
2) - 4*a^4*d^7*x*(b^2 - 4*a*c)^(1/2) - 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^
3*(b^2 - 4*a*c)^(1/2) + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 16*a^4*c*d^
6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/
2) - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x - 4*a^3*b^2
*d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a
*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*d^3*e^4 + 17
*a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x - 12*a^2*c^3
*d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) -
```

```
 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(
1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^5*x - 4*a^
2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^3*d*e
^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4
*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 32*a^3*b*c*d^5*e^2*x +
6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)
^(1/2))*(b^5*d^2 + b^4*d^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*e^2 + 8*a^2*b*c^2*
d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) -
 2*b^4*c*d*e - 6*a*b^3*c*d^2 - 4*a*b*c^3*e^2 - 8*a^2*c^3*d*e - 2*a*c^3*e^2*
(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2)
- 2*b^3*c*d*e*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*
(4*a^5*c*d^4 - a^4*b^2*d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*
e^4 - a^2*b^4*d^2*e^2 + 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d
*e^3 - 8*a^3*b*c^2*d*e^3 + 2*a^3*b^2*c*d^2*e^2)) - (log(c^4*e^7*(b^2 - 4*a*
c)^(1/2) - b*c^4*e^7 - 8*a^4*c*d^7 + 2*a^3*b^2*d^7 - b^5*d^4*e^3 - 3*a^2*b^
3*d^6*e + 4*b^2*c^3*d*e^6 + 4*b^4*c*d^3*e^4 + b^4*d^4*e^3*(b^2 - 4*a*c)^(1/
2) + 24*a^2*c^3*d^3*e^4 - 8*a^3*c^2*d^5*e^2 - 6*b^3*c^2*d^2*e^5 - 8*a*c^4*d
*e^6 - 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/2) - 4*a^4*d^7*x*(b^2 -
 4*a*c)^(1/2) + 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^3*(b^2 - 4*a*c)^(1/2) +
 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 16*a^4*c*d^6*e*x + 8*a^3*c*d^6*e*(
b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) + 18*a*b*c^3*d^2*e^5
 + 8*a*b^3*c*d^4*e^3 + 2*a*b^4*d^4*e^3*x + 4*a^3*b^2*d^6*e*x + 3*a^2*b^2*d^
6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 4*b^3*c*d^3
*e^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^4 - 17*a^2*b*c^2*d^4*e^3 + 2*
a^2*b^2*c*d^5*e^2 - 8*a^2*b^3*d^5*e^2*x + 12*a^2*c^3*d^2*e^5*x - 34*a^3*c^2
*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) - 18*a^2*b*c*d^5*e^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e^2*
x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^2*e^5*x + 4*a^2*b*c^2*d^3*e^4*x - 8*a
^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 4*a*b*c^3*d*e^6*x + 12*a^2*c^2*d^3*e
^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 4*a*c^3*d
*e^6*x*(b^2 - 4*a*c)^(1/2) + 32*a^3*b*c*d^5*e^2*x + 6*a*b*c^2*d^2*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2*(b^2 -
 4*a*c)^(1/2) - b^5*d^2 - b^3*c^2*e^2 - 8*a^2*b*c^2*d^2 + 2*a^2*c^2*d^2*(b^
2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e + 6*a*b^3*
c*d^2 + 4*a*b*c^3*e^2 + 8*a^2*c^3*d*e - 2*a*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 1
0*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^3*c*d*e*(b^2 - 4*
a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^4 - a^4*b^2*
d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*e^4 - a^2*b^4*d^2*e^2 +
 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d*e^3 - 8*a^3*b*c^2*d*e^
3 + 2*a^3*b^2*c*d^2*e^2)) - (a*d^4)/(e*(a*d*e^2 + a*e^3*x)*(a*d^2 + c*e^2 -
 b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=246

$$\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Rubi [A] time = 0.40, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de-2b^2cde+b^3d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{d^3}{e^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^3}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)}{e\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{cd(bd-2\cdots}{(ac\cdots}\right.$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \int \frac{cd(bd-2\cdots}{(\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2\cdots\right.}{\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2\cdots\right.}{\cdots}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3d^2-2b^2cde+4ac^2de-bc\left(3ad^2-ce^2\right)\right)\tan\cdots}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.23, size = 207, normalized size = 0.84

$$\frac{\frac{\left(c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\log(x(ax+b)+c)}{a} - \frac{2\left(bc\left(ce^2-3ad^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}} + \frac{2\log(d+ex)\left(ad^4+d^2e(3ce-2bd)\right)}{e^2} + \frac{2d^3\left(ad^2+e(ce-bd)\right)}{e^2(d+ex)}}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[
d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b
+ a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((a + c/x^2 + b/x)*(d + e*x)^2), x]
```

fricas [B] time = 56.31, size = 1465, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
- 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c -
2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^
2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqr
t(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*
c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 +
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2
*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 -
 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2
- 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*
d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3
*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d
^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*
x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2
*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2
*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2
*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)
/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b
*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x
^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e +
3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4
*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3
*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2
 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c
)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x)]
```

giac [**A**] time = 0.42, size = 412, normalized size = 1.67

$$\frac{1}{2}\left(\frac{2\,d^3e^2}{\left(ad^2e^3 - bde^4 + ce^5\right)(xe + d)} + \frac{2\left(b^3d^2e^3 - 3\,abcd^2e^3 - 2\,b^2cde^4 + 4\,ac^2de^4 + bc^2e^5\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^3d^4 - 2\,a^2bd^3e + ab^2d^2e^2 + 2\,a^2cd^2e^2 - 2\,abcde^3 + ac^2e^4\right)\sqrt{-b^2 + 4\,ac}} + \frac{\left(b^2d^2e - acd^2e - 2\,bcde^2 + c^2e^3\right)\log\left(-a + \frac{2\,ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{a^3d^4 - 2\,a^2bd^3e + ab^2d^2e^2 + 2\,a^2cd^2e^2 - 2\,abcde^3 + ac^2e^4} - \frac{2\,e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)}{a}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3*d^2*e^3 -
 3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*
d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)
/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a
^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e
- a*c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e
+ d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^4 -
2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4
) - 2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)*e^(-1)
```

maple [**B**] time = 0.01, size = 580, normalized size = 2.36

$$-\frac{b^3d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\,a} + \frac{2b^2cde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\,a} - \frac{b\,c^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}\,a} + \frac{3bc\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{4c^2de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}} + \frac{a\,d^4\ln(ex+d)}{\left(a\,d^2 - deb + c\,e^2\right)^2e^2} + \frac{b^2d^2\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)^2a} - \frac{bcde\ln\left(a\,x^2 + bx + c\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2a} + \frac{c^2e^2\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)^2a} - \frac{2b\,d^3\ln(ex+d)}{\left(a\,d^2 - deb + c\,e^2\right)^2e} + \frac{3c\,d^2\ln(ex+d)}{\left(a\,d^2 - deb + c\,e^2\right)^2} - \frac{c\,d^2\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)^2} + \frac{d^3}{\left(a\,d^2 - deb + c\,e^2\right)(ex+d)\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*d^2+1/2/(a*d^2-b*d*e+c*e^2)^2/
a*ln(a*x^2+b*x+c)*b^2*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d*e
+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*e^2+3/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2-4/(a*d^2-b*
d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*e-
1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))/a*b^3*d^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))/a*b^2*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))/a*b*c^2*e^2+d^4/(a*d^2-b*d*e+c*e^2)^2/e^2*ln
(e*x+d)*a-2*d^3/(a*d^2-b*d*e+c*e^2)^2/e*ln(e*x+d)*b+3*d^2/(a*d^2-b*d*e+c*e^
2)^2*ln(e*x+d)*c+1/e^2*d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.11, size = 2037, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 + b
^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (log(a^2*b^2*d^
6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2) +
 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 -
4*a*c)^(1/2) - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e + 4*b
*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b^2
 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d^5*e + 8*a*c
^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*
d^5*e*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2 - 6*a*b^
3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2
 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(b^
2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*c^2*d^3*e^3
*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*
c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2 -
 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b^2
 - 4*a*c)^(1/2) + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x + 23*a^2*b*c*d^
4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 + b
^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*
a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e - 3*a*b*c*d^2*(
b^2 - 4*a*c)^(1/2) + 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e^4
 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3 -
8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) - (log(2*c^4*e^
6 + 4*a^3*c*d^6 - a^2*b^2*d^6 + b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2)
 - 24*a*c^3*d^2*e^4 - 6*b^3*c*d^3*e^3 - 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2
- 4*a*c)^(1/2) + 10*a^2*c^2*d^4*e^2 + 9*b^2*c^2*d^2*e^4 + 2*a*b^3*d^5*e - 4
*b*c^3*d*e^5 + b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b
^2 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c*d^5*e - 8*a
*c^3*d*e^5*x + 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d^5*e*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^2*d^3*e^3 - 6*a*b^2*c*d^4*e^2 + 6*a*
```

```
b^3*d^4*e^2*x - 2*a^2*b^2*d^5*e*x + 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(
b^2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*c^2*d^3*e
^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*
a*c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2
 - 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b
^2 - 4*a*c)^(1/2) - 6*a*b*c^2*d^2*e^4*x - 2*a*b^2*c*d^3*e^3*x - 23*a^2*b*c*
d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 -
 b^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e -
5*a*b^2*c*d^2 - b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e + 3*a*b*c*d^2
*(b^2 - 4*a*c)^(1/2) - 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d*e*(b^2 -
 4*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e
^4 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3
- 8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) + d^3/(e^2*(d
 + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+\right.}{2\left(ad^2-e(bd-ce)\right)^2}$$

Rubi [A] time = 0.31, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^2}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{de(bd-2ce)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-c\left(ad^2-ce^2\right)-}{\left(ad^2-e(bd-ce)\right.}\right.$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{-c\left(ad^2-ce^2\right)-ad(bd-2ce)x}{c+bx+ax^2}\,d}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{(d(bd-2ce))\int \frac{b+2ax}{c+bx+ax^2}}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{d(bd-2ce)\log\left(c+bx\right.}{2\left(ad^2-e(bd-ce)\right.}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(b^2d^2-2bcde-2c\left(ad^2-ce^2\right)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.23, size = 159, normalized size = 0.82

$$\frac{\frac{2\left(2c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2d^2\left(ad^2+e(ce-bd)\right)}{e(d+ex)} - d(bd-2ce)\log(x(ax+b)+c) + 2d(bd-2ce)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*
e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b
^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b
 + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*(d + e*x)^2), x]
```

fricas [B] time = 19.72, size = 1120, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c
^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d
*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x
^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x +
c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*
c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*((b^3 -
 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 -
2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*
(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^
3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)
*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^
2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x), -1/2*(2*
(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e
^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 -
2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4
*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*
log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e
^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))
/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^
2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c
^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*
c^2 - 4*a*c^3)*e^6)*x)]
```

giac [A] time = 0.35, size = 331, normalized size = 1.71

$$\frac{\left(b^2d^2e^2 - 2\,acd^2e^2 - 2\,bcde^3 + 2\,c^2e^4\right)\arctan\left(\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)\sqrt{-b^2 + 4\,ac}} - \frac{d^2e}{\left(ad^2e^2 - bde^3 + ce^4\right)(xe + d)} - \frac{\left(bd^2 - 2\,cde\right)\log\left(a - \frac{2\,ad}{xe+d} + \frac{ad^2}{(xe+d)^2} + \frac{be}{xe+d} - \frac{bde}{(xe+d)^2} + \frac{ce^2}{(xe+d)^2}\right)}{2\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*arctan((2*a*d - 2*a
*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-
b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
- 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - d^2*e/((a*d^2*e^2 - b*d*e^3
+ c*e^4)*(x*e + d)) - 1/2*(b*d^2 - 2*c*d*e)*log(a - 2*a*d/(x*e + d) + a*d^2
/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*
d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)
```

maple [B] time = 0.01, size = 389, normalized size = 2.01

$$-\frac{2ac\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}+\frac{b^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}-\frac{2bcde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}+\frac{2c^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2-deb+c\,e^2\right)^2\sqrt{4ac-b^2}}+\frac{b\,d^2\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)^2}-\frac{b\,d^2\ln\left(a\,x^2+bx+c\right)}{2\left(a\,d^2-deb+c\,e^2\right)^2}-\frac{2cde\ln\left(ex+d\right)}{\left(a\,d^2-deb+c\,e^2\right)^2}+\frac{cde\ln\left(a\,x^2+bx+c\right)}{\left(a\,d^2-deb+c\,e^2\right)^2}-\frac{d^2}{\left(a\,d^2-deb+c\,e^2\right)\left(ex+d\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln
(a*x^2+b*x+c)*c*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2))*a*c*d^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e+2/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*e^2-d^2/e/(a*d^
2-b*d*e+c*e^2)/(e*x+d)+d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-2*d/(a*d^2-b*d
*e+c*e^2)^2*ln(e*x+d)*c*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.09, size = 1585, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^3
*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*
d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x
+ 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) - a^2*c*d^4*(b^2 - 4*a*c)^(1/2) - 6*a*b^2
*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x - 2*b*c^2*d*e^3*(b^2 - 4*a*c)^
(1/2) + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) - b*c^2*e^4*x*(b^2 - 4*a*c)^(1/2)
 + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^
2*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 28*a^2*c^2*d^2*
e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^2*d*e^3*x - 12*a^2*b*
c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) + 8*a*c^2*d*e^3*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e^3*x*(b^2 - 4
*a*c)^(1/2) + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*(d^2*(b^3/2 + (b^2*(b^
2 - 4*a*c)^(1/2))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e +
b*e*(b^2 - 4*a*c)^(1/2))) + c^2*(e^2*(b^2 - 4*a*c)^(1/2) + 4*a*d*e)))/(4*a^
3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2
*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*
e + 2*a*b^2*c*d^2*e^2) - (log(2*a*b^3*d^4 + b*c^3*e^4 + c^3*e^4*(b^2 - 4*a*
c)^(1/2) + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4
*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3
*c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) + a^2*c*d^4*(b^2
 - 4*a*c)^(1/2) - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x + 2*b
*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) + b*c^2*
e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2 - 14*a*c^2*d^2*e^2*(b^2 - 4*
a*c)^(1/2) - b^2*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*x*(b^2 - 4*a*c
)^(1/2) + 28*a^2*c^2*d^2*e^2*x + 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*
b*c^2*d*e^3*x - 12*a^2*b*c*d^3*e*x + 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) -
8*a*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) +
 2*b^2*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2
))*(c*(d^2*(2*a*b - a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e - b*e*(b^2 - 4*a*c)^(
1/2))) - d^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + c^2*(e^2*(b^2 - 4*a*c)
^(1/2) - 4*a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4
- b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c
^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) + (log(d + e*x)*(b*d^2 - 2*
c*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*
c*d^2*e^2) - d^2/(e*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)

[Out] Timed out
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal. Leaf size=183

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)1}{\left(ad^2-e(b\right.}$$

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \int \frac{x}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e\left(-ad^2+ce^2\right)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{ce(2ad-}{\left(ad^2-e(bd\right.}\right.$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{ce(2ad-be)+a\left(ad^2-ce^2\right)x}{c+bx+ax^2}\,d}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\int \frac{b+2ax}{c+bx+ax^2}}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\log\left(c+bx\right.}{2\left(ad^2-e(bd-ce\right.}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\left(bce^2+ad(bd-4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(ad^2-\right.}{\left(ad^2\right.}$$

Mathematica [A] time = 0.24, size = 148, normalized size = 0.81

$$\frac{-\frac{2\left(ad(bd-4ce)+bce^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \left(ad^2-ce^2\right)\log(x(ax+b)+c) + \frac{2d\left(ad^2+e(ce-bd)\right)}{d+ex} + \left(2ce^2-2ad^2\right)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*
e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2
+ 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x*(d + e*x)^2), x]
```

fricas [B] time = 16.70, size = 1059, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^
2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 +
b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b
^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x
)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2
 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2
*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^
2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4
+ ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^
3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2
*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4
*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a
*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 +
 ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c)
- 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d
^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2
*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3
*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*
d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

giac [A] time = 0.37, size = 323, normalized size = 1.77

$$-\frac{1}{2}\left(\frac{2\left(abd^2e - 4\,acde^2 + bce^3\right)\arctan\left(\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)\sqrt{-b^2+4\,ac}} - \frac{\left(ad^2 - ce^2\right)\log\left(a - \frac{2\,ad}{xe+d} + \frac{ad^2}{(xe+d)^2} + \frac{be}{xe+d} - \frac{bde}{(xe+d)^2} + \frac{ce^2}{(xe+d)^2}\right)}{a^2d^4e - 2\,abd^3e^2 + b^2d^2e^3 + 2\,acd^2e^3 - 2\,bcde^4 + c^2e^5} - \frac{2\,de}{\left(ad^2e^2 - bde^3 + ce^4\right)(xe+d)}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan((2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^
3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - (a*d^2 - c*e^2)*log(a - 2*a*d/(x*e + d)
+ a*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2
)/(a^2*d^4*e - 2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 +
c^2*e^5) - 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))*e
```

maple [A] time = 0.01, size = 328, normalized size = 1.79

$$-\frac{ab\,d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}} + \frac{4acde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{bc\,e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,d^2 - deb + c\,e^2\right)^2\sqrt{4ac-b^2}} - \frac{a\,d^2\ln(ex+d)}{\left(a\,d^2 - deb + c\,e^2\right)^2} + \frac{a\,d^2\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)^2} + \frac{c\,e^2\ln(ex+d)}{\left(a\,d^2 - deb + c\,e^2\right)^2} - \frac{c\,e^2\ln\left(a\,x^2 + bx + c\right)}{2\left(a\,d^2 - deb + c\,e^2\right)^2} + \frac{d}{\left(a\,d^2 - deb + c\,e^2\right)(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x)
```

```
[Out] 1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*d^2-1/2/(a*d^2-b*d*e+c*e^2)^2*l
n(a*x^2+b*x+c)*c*e^2-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*e^2+d/(a*d^2-b*d*e+c*e^2)
/(e*x+d)-1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln
(e*x+d)*c*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.07, size = 1768, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2*
d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 - 4
*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*
d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4*x +
 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3
*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d
^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*
c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*
e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 16*a*b^3*c^
2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e - 64*
a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a*b^5*d^2*e^
2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3*e*x - 6*a*b
^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 160*a^2*b^2*c^2*d*e^3*x - 56*a^2*b^3*c*d
^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(
b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*d^3*e*x)*(b^2*((a*d^2)/2 - (c*e^2)/2) - b
*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2) - 2*a^2*c*
d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*
e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3
*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*
e^2) - (log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*c^4*e^4 + 8*b^4*c^2*e^4
 - 56*a^3*b^2*c*d^4 - 56*a*b^2*c^3*e^4 + 4*a^3*b^3*d^4*x - 320*a^3*c^3*d^2*
e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e
^4*x*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2)
- 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*
d*e^3*x*(b^2 - 4*a*c)^(5/2) - 16*a^4*b*c*d^4*x + 112*a^2*b^2*c^2*d^2*e^2 -
8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*
a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3
*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 16*a*b^3*c^2*d*e^3 - 8*a*b^4*c*d^2*e^2 + 64*
a^2*b*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e + 64*a^3*b*c^2*d^3*e - 60*a*b^3*c^2*e^
4*x + 112*a^2*b*c^3*e^4*x - 4*a*b^5*d^2*e^2*x + 8*a^2*b^4*d^3*e*x - 256*a^3
*c^3*d*e^3*x + 256*a^4*c^2*d^3*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2)
+ 160*a^2*b^2*c^2*d*e^3*x + 56*a^2*b^3*c*d^2*e^2*x - 160*a^3*b*c^2*d^2*e^2*
x - 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(b^2 - 4*a*c)^(3/2) - 96*a^3*b^2*c
*d^3*e*x)*(b*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2
) + b^2*((a*d^2)/2 - (c*e^2)/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 - 2*a*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 -
 b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^
2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(a*d^2 - c*e
^2))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*c*d
^2*e^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+$$

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1569, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \int \frac{1}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\int \frac{ad-be-aex}{(d+ex)\left(c+bx+ax^2\right)}\,dx}{ad^2-bde+ce^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\int \left(\frac{e^2(2ad-be)}{\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right.}{ad^2-bde+ce^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(}{c+bx+ax^2}}{\left(ad^2-e(bd-ce\right.}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{(e(2ad-be))\int \frac{b+2ax}{c+bx+ax^2}}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{e(2ad-be)\log\left(c+bx\right.}{2\left(ad^2-e(bd-ce\right)}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(2a^2d^2+b^2e^2-2ae(bd+ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} +$$

Mathematica [A] time = 0.21, size = 151, normalized size = 0.80

$$\frac{\frac{2\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2e\left(ad^2+e(ce-bd)\right)}{d+ex} + e(be-2ad)\log(x(ax+b)+c) - 2e(be-2ad)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*
a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])
/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2), x]
```

fricas [B] time = 9.28, size = 1079, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a
*c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e -
 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a
*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + (2
*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*
d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^
4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3
 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^
2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c
^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e -
2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d^3 - 2*a*b*d^
2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)
) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*
c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a
^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4
*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b
*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d
^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3
- 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4
*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

giac [A] time = 0.35, size = 331, normalized size = 1.75

$$-\frac{\left(2\,a^{2}d^{2}e^{2}-2\,abde^{3}+b^{2}e^{4}-2\,ace^{4}\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^{2}}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^{2}}{xe+d}\right)e^{(-1)}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-2)}}{\left(a^{2}d^{4}-2\,abd^{3}e+b^{2}d^{2}e^{2}+2\,acd^{2}e^{2}-2\,bcde^{3}+c^{2}e^{4}\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(2\,ade-be^{2}\right)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^{2}}{(xe+d)^{2}}-\frac{be}{xe+d}+\frac{bde}{(xe+d)^{2}}-\frac{ce^{2}}{(xe+d)^{2}}\right)}{2\left(a^{2}d^{4}-2\,abd^{3}e+b^{2}d^{2}e^{2}+2\,acd^{2}e^{2}-2\,bcde^{3}+c^{2}e^{4}\right)}-\frac{e^{3}}{\left(ad^{2}e^{2}-bde^{3}+ce^{4}\right)(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*a*d - 2*a*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^
2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*a*d*e - b*e^2)*log(-a +
 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 -
c*e^2/(x*e + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2
*b*c*d*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d))
```

maple [B] time = 0.01, size = 386, normalized size = 2.04

$$\frac{2a^{2}d^{2}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}\sqrt{4ac-b^{2}}}-\frac{2abde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}\sqrt{4ac-b^{2}}}-\frac{2ac\,e^{2}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}\sqrt{4ac-b^{2}}}+\frac{b^{2}e^{2}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}\sqrt{4ac-b^{2}}}+\frac{2ade\ln(ex+d)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}}-\frac{ade\ln\left(a\,x^{2}+bx+c\right)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}}-\frac{b\,e^{2}\ln(ex+d)}{\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}}+\frac{b\,e^{2}\ln\left(a\,x^{2}+bx+c\right)}{2\left(a\,d^{2}-deb+c\,e^{2}\right)^{2}}-\frac{e}{\left(a\,d^{2}-deb+c\,e^{2}\right)(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x)
```

```
[Out] -1/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*ln
(a*x^2+b*x+c)*b*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2))*a^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*e^2+1/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2-e/(a*d^2-b*
d*e+c*e^2)/(e*x+d)+2*e/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d-e^2/(a*d^2-b*d*e
+c*e^2)^2*ln(e*x+d)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.11, size = 1782, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(c*e^4*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^2
 - 4*a*c)^(3/2) - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^(3/2) + 60*a*b^
3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*a^3
*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 10*b*
d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d^4
+ 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*b^2*c*e^4*(b^2 - 4*
a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^3*x*(b^2 - 4*a*c)^(
5/2) + 24*a*b^4*c*d*e^3 + 64*a*b^4*c*e^4*x + 32*a*b^5*d*e^3*x - 8*a^2*b*d^3
*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*
d^3*e + 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 160*a^2
*b^2*c^2*d*e^3 - 56*a^2*b^3*c*d^2*e^2 + 160*a^3*b*c^2*d^2*e^2 - 136*a^2*b^2
*c^2*e^4*x - 40*a^2*b^4*d^2*e^2*x - 448*a^4*c^2*d^2*e^2*x + 48*a^2*b*d^2*e^
2*x*(b^2 - 4*a*c)^(3/2) + 272*a^3*b^2*c*d^2*e^2*x - 64*a^4*b*c*d^3*e*x - 24
*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 240*a^2*b^3*c*d*e^3*x + 448*a^3*b*c^2*
d*e^3*x)*(a*(e^2*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d - b*d*(b^2 - 4*
a*c)^(1/2))) - e^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + a^2*(d^2*(b^2 -
4*a*c)^(1/2) - 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^
2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8
*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(b*e^
2 - 2*a*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3
+ 2*a*c*d^2*e^2) - (log(c*e^4*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4 + 8*b^6*e^4
*x - 4*a^3*d^4*(b^2 - 4*a*c)^(3/2) + 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a
*c)^(3/2) - 60*a*b^3*c^2*e^4 + 112*a^2*b*c^3*e^4 - 4*a*b^5*d^2*e^2 + 8*a^2*
b^4*d^3*e - 256*a^3*c^3*d*e^3 + 256*a^4*c^2*d^3*e + 8*a^4*b^2*d^4*x - 32*a^
3*c^3*e^4*x + 10*b*d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2
) - 16*a^4*b*c*d^4 - 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*
b^2*c*e^4*(b^2 - 4*a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^
3*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^4*c*d*e^3 - 64*a*b^4*c*e^4*x - 32*a*b^5*d*
e^3*x - 8*a^2*b*d^3*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3
```

```
/2) - 96*a^3*b^2*c*d^3*e - 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a
*c)^(3/2) + 160*a^2*b^2*c^2*d*e^3 + 56*a^2*b^3*c*d^2*e^2 - 160*a^3*b*c^2*d^
2*e^2 + 136*a^2*b^2*c^2*e^4*x + 40*a^2*b^4*d^2*e^2*x + 448*a^4*c^2*d^2*e^2*
x + 48*a^2*b*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 272*a^3*b^2*c*d^2*e^2*x + 64*a
^4*b*c*d^3*e*x - 24*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 240*a^2*b^3*c*d*e^3
*x - 448*a^3*b*c^2*d*e^3*x)*(e^2*(b^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2) - a*
(e^2*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d + b*d*(b^2 - 4*a*c)^(1/2)))
 + a^2*(d^2*(b^2 - 4*a*c)^(1/2) + 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a
^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e
+ 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) -
e/((d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{}{d(}$$

Rubi [A] time = 0.41, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{e^2}{d(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \int \frac{1}{x(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x} + \frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e^3\left(-3ad^2+e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2(d+ex)}\right.$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\tanh}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.25, size = 246, normalized size = 0.99

$$\frac{\left(-a^2d^2+ae(2bd+ce)-b^2e^2\right)\log(x(ax+b)+c)}{2c\left(ad^2+e(ce-bd)\right)^2} - \frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{e^2}{d(d+ex)\left(ad^2+e(ce-bd)\right)} - \frac{e^2\log(d+ex)\left(3ad^2+e(ce-2bd)\right)}{\left(ad^3+de(ce-bd)\right)^2} + \frac{\log(x)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3
*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqr
t[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*
d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(
a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 391, normalized size = 1.58

$$-\frac{\left(a^{2} b d^{2} e^{2}-2 a b^{2} d e^{3}+4 a^{2} c d e^{3}+b^{3} e^{4}-3 a b c e^{4}\right) \arctan\left(\frac{\left(2 a d-\frac{2 a d^{2}}{x e+d}-b e+\frac{2 b d e}{x e+d}-\frac{2 c e^{2}}{x e+d}\right) e^{(-1)}}{\sqrt{-b^{2}+4 a c}}\right) e^{(-2)}}{\left(a^{2} c d^{4}-2 a b c d^{3} e+b^{2} c d^{2} e^{2}+2 a c^{2} d^{2} e^{2}-2 b c^{2} d e^{3}+c^{3} e^{4}\right) \sqrt{-b^{2}+4 a c}}-\frac{\left(a^{2} d^{2}-2 a b d e+b^{2} e^{2}-a c e^{2}\right) \log\left(a-\frac{2 a d}{x e+d}+\frac{a d^{2}}{(x e+d)^{2}}+\frac{b e}{x e+d}-\frac{b d e}{(x e+d)^{2}}+\frac{c e^{2}}{(x e+d)^{2}}\right)}{2\left(a^{2} c d^{4}-2 a b c d^{3} e+b^{2} c d^{2} e^{2}+2 a c^{2} d^{2} e^{2}-2 b c^{2} d e^{3}+c^{3} e^{4}\right)}+\frac{e^{5}}{\left(a d^{3} e^{3}-b d^{2} e^{4}+c d e^{5}\right)(x e+d)}+\frac{\log\left(\left|-\frac{d}{x e+d}+1\right|\right)}{c d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*ar
ctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e +
d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d
^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)) - 1
/2*(a^2*d^2 - 2*a*b*d*e + b^2*e^2 - a*c*e^2)*log(a - 2*a*d/(x*e + d) + a*d^
2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2
*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 +
c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d)) + log(abs(-d/(
x*e + d) + 1))/(c*d^2)
```

maple [B] time = 0.01, size = 589, normalized size = 2.38

$$-\frac{a^{2} b d^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c}-\frac{4 a^{2} d e \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}}+\frac{2 a b^{2} d e \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c}+\frac{3 a b e^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}}-\frac{b^{3} e^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c}-\frac{a^{2} d^{2} \ln\left(a x^{2}+b x+c\right)}{2\left(a d^{2}-d e b+c e^{2}\right)^{2} c}+\frac{a b d e \ln\left(a x^{2}+b x+c\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} c}-\frac{3 a e^{2} \ln(e x+d)}{\left(a d^{2}-d e b+c e^{2}\right)^{2}}+\frac{a e^{2} \ln\left(a x^{2}+b x+c\right)}{2\left(a d^{2}-d e b+c e^{2}\right)^{2}}-\frac{b^{2} e^{2} \ln\left(a x^{2}+b x+c\right)}{2\left(a d^{2}-d e b+c e^{2}\right)^{2} c}+\frac{2 b e^{3} \ln(e x+d)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} d}-\frac{c e^{4} \ln(e x+d)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} d^{2}}+\frac{e^{2}}{\left(a d^{2}-d e b+c e^{2}\right)(e x+d) d}+\frac{\ln(x)}{c d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x^2+b*x+c)*d^2+1/(a*d^2-b*d*e+c*e^2)^
2/c*a*ln(a*x^2+b*x+c)*b*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*e^2
-1/2/(a*d^2-b*d*e+c*e^2)^2/c*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e^2)^
2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d^2-4/(a*d^
2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*
d*e+2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2
)^(1/2))*a*b^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*a*b*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2+ln(x)/c/d^2+e^2/d/(a*d^2-b*d*e+c
*e^2)/(e*x+d)-3*e^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a+2*e^3/(a*d^2-b*d*e+c*
e^2)^2/d*ln(e*x+d)*b-e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 25.28, size = 3510, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4)/(d*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*
e^2 - b*d*e)^2) - (((a*e^3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d
^2*e^2 - 7*a^2*b^2*d^3*e + 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e
- a*b^2*c*d*e^3 - 8*a^2*b*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((
a*e*(a^3*b*d^5 - 4*a*c^3*e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2
- 3*a^2*b^2*d^4*e - 8*a^2*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b
*c^2*d*e^4 + 6*a*b^2*c*d^2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d
*e^2) + (a*e*x*(3*a^4*d^5 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4
*a^2*c^2*d*e^4 + 19*a^3*c*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a
^3*b*d^4*e + 15*a*b^2*c*d*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d
*e^2) - (a*e*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2
*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c
)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^
2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*
c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e
^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^
2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c
^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x
))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 +
b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5
*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*
(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 -
4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*
a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4
*a*b^2*c*e^4 - 4*a*b^3*d*e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2
- 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2
+ a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)
^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))
)/(2*(4*a*c^4*e^4 + 4*a^3*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2
*d*e^3 - b^4*c*d^2*e^2 + 8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^
3*e - 8*a^2*b*c^2*d^3*e + 2*a*b^2*c^2*d^2*e^2)) + (log((a^4*e^4)/(d*(a*d^2
+ c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e^
3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d^2*e^2 - 7*a^2*b^2*d^3*e
+ 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e - a*b^2*c*d*e^3 - 8*a^2*b
*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(a^3*b*d^5 - 4*a*c^3*
e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2 - 3*a^2*b^2*d^4*e - 8*a^2
*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b*c^2*d*e^4 + 6*a*b^2*c*d^
2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d*e^2) + (a*e*x*(3*a^4*d^5
 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4*a^2*c^2*d*e^4 + 19*a^3*c
*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a^3*b*d^4*e + 15*a*b^2*c*d
*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d*e^2) - (a*e*(b^4*e^2 - 4
*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*
a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e +
 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2
 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*
c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e
*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*
d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c*(4*a*c - b^2)*(a*
```

```
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*a^4*d^4 + b^4*e^4 + 2*a^2
*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*a*b^3*d*
e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2))*
(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*
c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a
^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1
/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 -
 b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*
d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)
^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2
 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^4 + 4*a^3
*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2*d*e^3 - b^4*c*d^2*e^2 +
8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e - 8*a^2*b*c^2*d^3*e +
 2*a*b^2*c^2*d^2*e^2)) - (log(d + e*x)*(c*e^4 + 3*a*d^2*e^2 - 2*b*d*e^3))/(
a^2*d^6 + b^2*d^4*e^2 + c^2*d^2*e^4 - 2*a*b*d^5*e + 2*a*c*d^4*e^2 - 2*b*c*d
^3*e^3) + log(x)/(c*d^2) + e^2/(d*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
[Out] Timed out
```

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal. Leaf size=291

$$\frac{\left(2a^3cd^2-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+\right.}{2c^2\left(ad^2-\right.}$$

Rubi [A] time = 0.56, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2a^3cd^2+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^
2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^
2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(
3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e
)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

```
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \int \frac{1}{x^2(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^2} + \frac{-bd-2ce}{c^2d^3x} + \frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{e^4\left(4ad^2-e(3bd\cdots\right.}{d^3\left(ad^2-e(bd-ce)\right)\cdots}\cdots\right.$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd\cdots\right.}{d^3\left(ad^2-e\cdots\right.}\cdots$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd\cdots\right.}{d^3\left(ad^2-e\cdots\right.}\cdots$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd\cdots\right.}{d^3\left(ad^2-e\cdots\right.}\cdots$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2\cdots\right.}{c^2\sqrt{b^2-4ac}\left(\cdots\right.}\cdots$$

Mathematica [A] time = 0.34, size = 287, normalized size = 0.99

$$\frac{\left(-2a^3cd^2+a^2\left(b^2d^2+6bcde+2c^2e^2\right)-2ab^2e(bd+2ce)+b^4e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\left(ad^2+e(ce-bd)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{e^3\log(d+ex)\left(4ad^2+e(2ce-3bd)\right)}{d^3\left(ad^2+e(ce-bd)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*
c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^
2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d
^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) +
 ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
 + e*(-(b*d) + c*e))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 487, normalized size = 1.67

$$-\frac{\left(a^2b^2d^2e^2 - 2a^3cd^2e^2 - 2ab^3de^3 + 6a^2bcde^3 + b^4e^4 - 4ab^2ce^4 + 2a^2c^2e^4\right)\arctan\left(-\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d} - \frac{2ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}\right)e^{(-2)}}{\left(a^2c^2d^4 - 2abc^2d^3e + b^2c^2d^2e^2 + 2ac^3d^2e^2 - 2bc^3de^3 + c^4e^4\right)\sqrt{-b^2+4ac}} + \frac{\left(a^2bd^2 - 2ab^2de + 2a^2cde + b^3e^2 - 2abce^2\right)\log\left(-a + \frac{2ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{2\left(a^2c^2d^4 - 2abc^2d^3e + b^2c^2d^2e^2 + 2ac^3d^2e^2 - 2bc^3de^3 + c^4e^4\right)} - \frac{e^7}{\left(ad^4e^4 - bd^3e^5 + cd^2e^6\right)(xe+d)} - \frac{\left(bde + 2ce^2\right)e^{(-1)}\log\left(\left|-\frac{d}{xe+d}+1\right|\right)}{c^2d^3} + \frac{e}{cd^3\left(\frac{d}{xe+d}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6*a^2*b*c*d*e^3 + b^4
*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) -
b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(
-2)/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2
*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2*b*d^2 - 2*a*b^2*d*e
+ 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*
e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^2*
d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 +
 c^4*e^4) - e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(x*e + d)) - (b*d*e +
2*c*e^2)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^2*d^3) + e/(c*d^3*(d/(x*e + d
) - 1))
```

maple [B] time = 0.01, size = 791, normalized size = 2.72

$$\frac{a^2bd^2\ln\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)^2c^2} + \frac{a^2de\ln\left(ax^2+bx+c\right)}{\left(ad^2-bde+ce^2\right)^2c} - \frac{ab^2de\ln\left(ax^2+bx+c\right)}{\left(ad^2-bde+ce^2\right)^2c^2} - \frac{abe^2\ln\left(ax^2+bx+c\right)}{\left(ad^2-bde+ce^2\right)^2c} + \frac{b^3e^2\ln\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)^2c^2} - \frac{2a^3d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c} + \frac{a^2b^2d^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c^2} + \frac{6a^2bde\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c} + \frac{2a^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}} - \frac{2ab^3de\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c^2} - \frac{4ab^2e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c} + \frac{b^4e^2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\left(ad^2-bde+ce^2\right)^2\sqrt{4ac-b^2}\,c^2} - \frac{1}{cd^2x} - \frac{b\ln(x)}{c^2d^2} - \frac{2e\ln(x)}{cd^3} - \frac{e^3}{\left(ad^2-bde+ce^2\right)d^2(ex+d)} + \frac{4e^3a\ln(ex+d)}{\left(ad^2-bde+ce^2\right)^2d} - \frac{3e^4b\ln(ex+d)}{\left(ad^2-bde+ce^2\right)^2d^2} + \frac{2e^5c\ln(ex+d)}{\left(ad^2-bde+ce^2\right)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x)
```

```
[Out] 1/2/c^2/(a*d^2-b*d*e+c*e^2)^2*a^2*ln(a*x^2+b*x+c)*b*d^2+1/c/(a*d^2-b*d*e+c*
e^2)^2*a^2*ln(a*x^2+b*x+c)*d*e-1/c^2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c
)*b^2*d*e-1/c/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*b*e^2+1/2/c^2/(a*d^2-
b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*b^3*e^2-2/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d^2+1/c^2/(a*d^2-b*d*e+c*e^
2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2*d^2+6/c/
(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*a^2*b*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*a^2*e^2-2/c^2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan(
(2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d*e-4/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2+1/c^2/(a*d^2-b*d*e+c*
e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2-1/c/d^
2/x-1/c^2/d^2*ln(x)*b-2/c/d^3*ln(x)*e-e^3/(a*d^2-b*d*e+c*e^2)/d^2/(e*x+d)+4
*e^3/(a*d^2-b*d*e+c*e^2)^2/d*ln(e*x+d)*a-3*e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln
(e*x+d)*b+2*e^5/(a*d^2-b*d*e+c*e^2)^2/d^3*ln(e*x+d)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 31.16, size = 4948, normalized size = 17.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2 +
 c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) + (x
*(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x + e*x
^2) - (log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^
4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 +
a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 -
12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7
 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^
3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (
((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^5*d^3*e^3
- 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*e^4 + 8*a^2
*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*d^5*e - 7*a*
b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 12*a^2*b^2*c*d
^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^4*e^2*(b^2 - 4
*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/
2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*
b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*
d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) +
 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*
c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d
^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*
b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8
*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c
 - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (2*a*e*x*(a*d - b*e)*(a^3*b*d^5 + 8*a*
c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 - a^2*b^2*d^4*e + 16*
a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b*c^2*d*e^4 - 8*a*b^2
*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e^2
+ b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2
*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*
a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/
2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^
2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)
*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c^4*e^8 + 4*b^4*c^2*e
^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 + 2*a^5*c*d^6*e^2 + a
^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 + 18*a^4*c^2*d^4*e^4 -
 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6 + 8*a*b^3*c^2*d*e^7
 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d^5*e^3 + 10*a^2*b^3*
c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2))*(b^
5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b
^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*
e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*
c)^(1/2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d
*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e
^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d - b*
e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*
```

```
a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e
^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*d*e - 4
*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*
b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*e^4
 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e^2
- 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*d^2
*e^2)) + (log((a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^
4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*
e^5 - 2*a*b^5*d^4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^
7 - b^5*c*d^2*e^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5
+ a^4*c^2*d^5*e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5
 + 4*a*b^2*c^3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*
c^3*d^2*e^6 + a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c
*e^2 - b*d*e)^2) - (((a*e*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*
d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^
2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*
d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 -
4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*
d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6
*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*
d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*
a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2
 - b*d*e)^2) - (a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6
+ b^5*d^3*e^3 - 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d
^2*e^4 + 8*a^2*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*
c*d^5*e - 7*a*b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 +
12*a^2*b^2*c*d^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (2*a*e*x*(a*d - b*
e)*(a^3*b*d^5 + 8*a*c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 -
 a^2*b^2*d^4*e + 16*a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b
*c^2*d*e^4 - 8*a*b^2*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2
- b*d*e)))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c
^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2
) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d
^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/
2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/
(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c
^4*e^8 + 4*b^4*c^2*e^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 +
 2*a^5*c*d^6*e^2 + a^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 +
18*a^4*c^2*d^4*e^4 - 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6
 + 8*a*b^3*c^2*d*e^7 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d
^5*e^3 + 10*a^2*b^3*c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*
e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a
^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c
)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a
^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*
c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1
/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d -
 b*e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) -
b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) +
 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*
c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e
- 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a
^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*
e^4 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e
^2 - 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*
d^2*e^2)) - (log(x)*(b*d + 2*c*e))/(c^2*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.70 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal. Leaf size=372

$$\frac{\left(a^3cd^2-a^2\left(b^2d^2+4bcde+c^2e^2\right)+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-a^3cd(3bd+4ce)+a^2b\right.}{}$$

Rubi [A] time = 0.85, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+4bcde+c^2e^2\right)+a^3cd^2+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+b^5e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)\left(-c\left(ad^2-3ce^2\right)+b^2d^2+2bcde\right)}{c^3d^4}+\frac{e^4}{d^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{e^4\log(d+ex)\left(5ad^2-e(4bd-3ce)\right)}{d^4\left(ad^2-e(bd-ce)\right)^2}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d - c
*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d + 5*
c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^
2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2
+ 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4
*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2
- b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2))*
Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \int \frac{1}{x^3(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^3} + \frac{-bd-2ce}{c^2d^3x^2} + \frac{b^2d^2+2bcde-c\left(ad^2-3ce^2\right)}{c^3d^4x} + \frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right.}\right.$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-\right.\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^5e^2-a^3cd(3bd+4ce)\right.}{}$$

Mathematica [A] time = 0.43, size = 370, normalized size = 0.99

$$-\frac{\left(-a^3cd^2+a^2\left(b^2d^2+4bcde+c^2e^2\right)-ab^2e(2bd+3ce)+b^4e^2\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)^2}+\frac{\left(a^3cd(3bd+4ce)-a^2b\left(b^2d^2+8bcde+5c^2e^2\right)+ab^3e(2bd+5ce)+b^5\left(-e^2\right)\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{\log(x)\left(c\left(3ce^2-ad^2\right)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2+e(3ce-4bd)\right)}{d^4\left(ad^2+e(ce-bd)\right)^2}+\frac{e^4}{d^3(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(2*
b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2)
+ ((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*
(5*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e)
)^2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2 +
4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e
))^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.49, size = 587, normalized size = 1.58

$$\frac{\left(a^{2} b^{3} d^{2} e^{2} - 3 \, a^{3} b c d^{2} e^{2} - 2 \, a b^{4} d e^{3} + 8 \, a^{2} b^{2} c d e^{3} - 4 \, a^{3} c^{2} d e^{3} + b^{5} e^{4} - 5 \, a b^{3} c e^{4} + 5 \, a^{2} b c^{2} e^{4}\right) \arctan\left(-\frac{\left(2 \, a d - \frac{2 \, a d^{2}}{x e + d} - b e + \frac{2 \, b d e}{x e + d} - \frac{2 \, c e^{2}}{x e + d}\right) e^{\left(-1\right)}}{\sqrt{-b^{2} + 4 \, a c}}\right) e^{\left(-2\right)}}{\left(a^{2} c^{3} d^{4} - 2 \, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2 \, a c^{4} d^{2} e^{2} - 2 \, b c^{4} d e^{3} + c^{5} e^{4}\right) \sqrt{-b^{2} + 4 \, a c}} - \frac{\left(a^{2} b^{2} d^{2} - a^{3} c d^{2} - 2 \, a b^{3} d e + 4 \, a^{2} b c d e + b^{4} e^{2} - 3 \, a b^{2} c e^{2} + a^{2} c^{2} e^{2}\right) \log\left(-a + \frac{2 \, a d}{x e + d} - \frac{a d^{2}}{{\left(x e + d\right)}^{2}} - \frac{b e}{x e + d} + \frac{b d e}{{\left(x e + d\right)}^{2}} - \frac{c e^{2}}{{\left(x e + d\right)}^{2}}\right)}{2 \, {\left(a^{2} c^{3} d^{4} - 2 \, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2 \, a c^{4} d^{2} e^{2} - 2 \, b c^{4} d e^{3} + c^{5} e^{4}\right)}} + \frac{e^{9}}{{\left(a d^{5} e^{5} - b d^{4} e^{6} + c d^{3} e^{7}\right)} {\left(x e + d\right)}} + \frac{{\left(b^{2} d^{2} e - a c d^{2} e + 2 \, b c d e^{2} + 3 \, c^{2} e^{3}\right)} e^{\left(-1\right)} \log\left({\left| -\frac{d}{x e + d} + 1 \right|}\right)}{c^{3} d^{4}} + \frac{2 \, b c d e + 5 \, c^{2} e^{2} - \frac{2 \, {\left(b c d^{2} e^{2} + 3 \, c^{2} d e^{3}\right)} e^{\left(-1\right)}}{x e + d}}{2 \, c^{3} d^{4} {\left(\frac{d}{x e + d} - 1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (a^2*b^3*d^2*e^2 - 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 -
4*a^3*c^2*d*e^3 + b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d
 - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/
sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^
2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a
^2*b^2*d^2 - a^3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*
e^2 + a^2*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e
+ d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*
e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + e^9/((a*
d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(x*e + d)) + (b^2*d^2*e - a*c*d^2*e + 2*b*
c*d*e^2 + 3*c^2*e^3)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^3*d^4) + 1/2*(2*b
*c*d*e + 5*c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)*e^(-1)/(x*e + d))/(c^3*d
^4*(d/(x*e + d) - 1)^2)
```

maple [B] time = 0.02, size = 993, normalized size = 2.67

$$\frac{3 a^{3} b d^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{2}}+\frac{4 a^{3} d e \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c}-\frac{a^{2} b^{3} d^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{3}}-\frac{8 a^{2} b^{2} d e \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{2}}-\frac{5 a^{2} b e^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c}+\frac{2 a b^{4} d e \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{3}}+\frac{5 a b^{3} e^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{2}}-\frac{b^{5} e^{2} \arctan\left(\frac{2 a x+b}{\sqrt{4 a c-b^{2}}}\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} \sqrt{4 a c-b^{2}}\, c^{3}}+\frac{a^{3} d^{2} \ln\left(a x^{2}+b x+c\right)}{2 \left(a d^{2}-d e b+c e^{2}\right)^{2} c^{2}}-\frac{a^{2} b^{2} d^{2} \ln\left(a x^{2}+b x+c\right)}{2 \left(a d^{2}-d e b+c e^{2}\right)^{2} c^{3}}-\frac{2 a^{2} b d e \ln\left(a x^{2}+b x+c\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} c^{2}}-\frac{a^{2} e^{2} \ln\left(a x^{2}+b x+c\right)}{2 \left(a d^{2}-d e b+c e^{2}\right)^{2} c}+\frac{a b^{3} d e \ln\left(a x^{2}+b x+c\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} c^{3}}+\frac{3 a b^{2} e^{2} \ln\left(a x^{2}+b x+c\right)}{2 \left(a d^{2}-d e b+c e^{2}\right)^{2} c^{2}}-\frac{5 a e^{4} \ln\left(e x+d\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} d^{2}}-\frac{b^{4} e^{2} \ln\left(a x^{2}+b x+c\right)}{2 \left(a d^{2}-d e b+c e^{2}\right)^{2} c^{3}}+\frac{4 b e^{5} \ln\left(e x+d\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} d^{3}}-\frac{3 c e^{6} \ln\left(e x+d\right)}{\left(a d^{2}-d e b+c e^{2}\right)^{2} d^{4}}+\frac{e^{4}}{\left(a d^{2}-d e b+c e^{2}\right) \left(e x+d\right) d^{3}}-\frac{a \ln\left(x\right)}{c^{2} d^{2}}+\frac{b^{2} \ln\left(x\right)}{c^{3} d^{2}}+\frac{2 b e \ln\left(x\right)}{c^{2} d^{3}}+\frac{3 e^{2} \ln\left(x\right)}{c \,d^{4}}+\frac{b}{c^{2} d^{2} x}+\frac{2 e}{c \,d^{3} x}-\frac{1}{2 c \,d^{2} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x)
```

```
[Out] -8/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*a^2*b^2*d*e-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(1/2)*arctan((2
*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3*d^2-5/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^2+5/(a*d^2-b*d*e+c*e^2)
^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^2-2/(a
*d^2-b*d*e+c*e^2)^2/c^2*a^2*ln(a*x^2+b*x+c)*b*d*e+1/(a*d^2-b*d*e+c*e^2)^2/c
^3*a*ln(a*x^2+b*x+c)*b^3*d*e+3/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/c/(4*
a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d*e-1/c^2/d^2*ln(x)*
a+1/c^3/d^2*ln(x)*b^2+3/c/d^4*ln(x)*e^2+1/c^2/d^2/x*b+2/c/d^3/x*e+e^4/(a*d^
2-b*d*e+c*e^2)/d^3/(e*x+d)+2/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a
^3*ln(a*x^2+b*x+c)*d^2-1/2/c/d^2/x^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x
^2+b*x+c)*e^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c^3*ln(a*x^2+b*x+c)*b^4*e^2+2/c^2/d
^3*ln(x)*b*e-5*e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*ln(e*x+d)*a+4*e^5/(a*d^2-b*d*e
+c*e^2)^2/d^3*ln(e*x+d)*b-3*e^6/(a*d^2-b*d*e+c*e^2)^2/d^4*ln(e*x+d)*c-1/2/(
a*d^2-b*d*e+c*e^2)^2/c^3*a^2*ln(a*x^2+b*x+c)*b^2*d^2+3/2/(a*d^2-b*d*e+c*e^2
)^2/c^2*a*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 45.61, size = 7144, normalized size = 19.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] ((x*(2*b*d + 3*c*e))/(2*c^2*d^2) - 1/(2*c*d) + (x^2*(3*c^2*e^4 - b^2*d^2*e^
2 + a*b*d^3*e - b*c*d*e^3 + 2*a*c*d^2*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e
)))/(d*x^2 + e*x^3) - (log(d + e*x)*(3*c*e^6 + 5*a*d^2*e^4 - 4*b*d*e^5))/(a
^2*d^8 + b^2*d^6*e^2 + c^2*d^4*e^4 - 2*a*b*d^7*e + 2*a*c*d^6*e^2 - 2*b*c*d^
5*e^3) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6
*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4
*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 +
 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c
^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3
*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2
*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 +
 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*
c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7
*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a
^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(12*a*c^5*e^
7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4
*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^
5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 +
 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*
c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5
*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(4*a
^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*
e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a
*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^
2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x -
6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 +
 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c
)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d
*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(
b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2
- 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (a*e*x*(
2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e
^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 - 5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 +
 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 - 24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e
- 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b
^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2
*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^
2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e
^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a
^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^
```

3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x*(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 - 85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 - 42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 - 6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 + 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e - 12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 + 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 + 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(12*a*c^5*e^7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 + 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2

```
 - 4*a*c)^(1/2) + 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 -
4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a
*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d
^2 + c*e^2 - b*d*e)^2) - (a*e*x*(2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^
7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 -
5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 + 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 -
24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e - 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*
e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24
*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*
(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*
d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 -
4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^
3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*
e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e
^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*
(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x*
(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a
^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2
*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*
e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 -
85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 -
42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 -
6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^
9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 +
 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5
*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^
2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2
 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a
*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c
*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e^2
*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*
(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e
^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4
 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e -
12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^
2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c
^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d
^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4
*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3
*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e
^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^
2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(
b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*
c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c
^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) +
(log(x)*(3*c^2*e^2 - d^2*(a*c - b^2) + 2*b*c*d*e))/(c^3*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

3.71 $\int (b+2cx) \left(a+bx+cx^2\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$\frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx) \left(a+bx+cx^2\right)^{13} \, dx = \frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 12.56

$$\frac{1}{14}x(b+cx)\left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7+2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9+364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}+14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 10
01*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)
^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b
+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^
11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b+2cx) \left(a+bx+cx^2\right)^{13} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)*(a + b*x + c*x^2)^13, x]
```

fricas [B] time = 0.76, size = 1446, normalized size = 90.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + x^26*c^13*a + 26*x^25*c
^11*b^3 + 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 + 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 + 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 + 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 + 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 + 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
3*a^2 + 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 + 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 + 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 + 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 + 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 + 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 + 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 + 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 + 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 + 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 + 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 + 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 + 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 + 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 + 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 +
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 + 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 + 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 + 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 + 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 + 1716/7*x^14*c^7*a^7 + x^13*b^13*a + 78*x^13*c*
b^11*a^2 + 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 + 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 + 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
+ 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 + 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 + 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 + 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 + 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 + 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
+ 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 + 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 + 143*x^10*c^5*a^9 + 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 + 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 + 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 + 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 + 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 + 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 + 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 + 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 + 26*x^6*c^3*a^11 + 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 + 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 + 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 + 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 +
x^2*c*a^13 + x*b*a^13
```

giac [B] time = 0.43, size = 216, normalized size = 13.50

$$\frac{1}{14}\left(cx^2+bx\right)^{14}+\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2+26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4+143\left(cx^2+bx\right)^{9}a^5+\frac{429}{2}\left(cx^2+bx\right)^{8}a^6+\frac{1716}{7}\left(cx^2+bx\right)^{7}a^7+\frac{429}{2}\left(cx^2+bx\right)^{6}a^8+143\left(cx^2+bx\right)^{5}a^9+\frac{143}{2}\left(cx^2+bx\right)^{4}a^{10}+26\left(cx^2+bx\right)^{3}a^{11}+\frac{13}{2}\left(cx^2+bx\right)^{2}a^{12}+\left(cx^2+bx\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13
```

maple [B] time = 0.00, size = 46548, normalized size = 2909.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.43, size = 14, normalized size = 0.88

$$\frac{1}{14}\left(cx^2 + bx + a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

mupad [B] time = 3.34, size = 1203, normalized size = 75.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^13*(a*b^13 + 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^15*(b^13*c + 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*
b^2*c^2) + x^22*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^18*(143*a^5*c^9 + (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 +
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 + 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 + x^2*(a^13*c + (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 + 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + b
*c^13*x^27 + (c^12*x^26*(2*a*c + 13*b^2))/2 + a^13*b*x + (143*a^7*b*x^7*(12
*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 13*a^3*b*x
^11*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 + 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 +
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c) + 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 + 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 + 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c + 2*b^2) + 13*b*c^11*x^25*(a*c + 2*b^2)
```

sympy [B] time = 0.35, size = 1326, normalized size = 82.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12
/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a*
b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10
+ 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4*
c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1
287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b
```

```
**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x*
*19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b
**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 +
8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c*
*4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**
6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**
6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17
16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b*
*7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a*
*7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c*
*4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) +
 x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85
80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1
2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a**
5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) +
 x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514
8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5
+ 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a*
*10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c
 + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a
**8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2
+ 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b*
*3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10
*b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b
**2/2)
```

3.72 $\int x(b+2cx^2)(a+bx^2+cx^4)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Rubi [A] time = 0.33, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}dx,x,x^2\right)\\ &= \frac{1}{28}\left(a+bx^2+cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(14a^{13}+91a^{12}x^2\left(b+cx^2\right)+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3+2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5+3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7+2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9+364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}+14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*x
^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8
*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^
2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*
x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12
 + x^26*(b + c*x^2)^13))/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] IntegrateAlgebraic[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13, x]
```

fricas [B] time = 0.76, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 + 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 + 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 + 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 + 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 + 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 + 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 + 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 + 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 + 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 + 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 +
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 + 1287/2
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 + 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 + 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 + 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 + 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
+ 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 + 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 + 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 + 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 + 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 + 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 +
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 + 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 + 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 + 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 + 858/7*x^28*c^7*a
^7 + 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 + 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 + 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 + 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 + 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 + 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 + 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 + 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 + 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 + 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 + 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 + 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 + 143/2*x^20*c^5*a^
9 + 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 + 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 + 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 + 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 + 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 + 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 + 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 + 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 + 13*x^12*c^3*a^11 + 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
+ 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 + 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 + 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 + 1/2*x^4*
c*a^13 + 1/2*x^2*b*a^13
```

giac [B] time = 0.66, size = 246, normalized size = 13.67

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}+\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2+13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4+\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6+\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8+\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}+13\left(cx^4+bx^2\right)^{3}a^{11}+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}+\frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 + 1/2*(c*x^4 + b*x^2)*a^13
```

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.49, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

mupad [B] time = 3.23, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 + 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 + 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 + 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 + 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
+ 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 + 2574*a^5*b^2*c^7) + x^26*((a*b
^13)/2 + 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 + 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 + 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 + 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 + 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 + 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 + 13*a^11*c^3 + (715*a^9*b^4*c)/2
```

```
+ (429*a^10*b^2*c^2)/2) + x^44*(13*a^3*c^11 + (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 + (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 + (143*a^9*c^5)/2 +
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 + 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) + x^36*((143*a^5*c^9)/2 + (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 + 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 +
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 + 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 + 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 + 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 + 858*a*b^6*c^7
 + (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^14*x^56)/28 + x^4*((a^13*c)/
2 + (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 + (c^12*x^52*(2*a*c + 13*b^2))/4 + (143*a^7*b*x^14*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (13*a^3*b*x^2
2*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/2 + (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c + 2*b^2))/2 + (13*b*c^11*x^50*(a*c + 2*b^2
))/2
```

sympy [B] time = 0.34, size = 1384, normalized size = 76.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b
**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**1
2/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*
b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/
2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a*
*2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
0/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b*
*8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**
5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435
*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8
*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c*
*7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**
11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13
*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b
**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**
13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**
5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 +
13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 +
 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a**2*b
**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**2*c**5 + 15015*
a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 + 143*a**3*b
**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 + 4290*a**7*b**3*c**4
 + 6006*a**6*b**5*c**3 + 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*
b**11) + x**20*(143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 858*a**6*b*
*7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715*a**9*b**2*c**3 + 64
35*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14*(143*a**10
```

```
*b*c**3 + 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**1
2*(13*a**11*c**3 + 429*a**10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**
6/4) + x**10*(39*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*
(13*a**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**12*b*c
/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

3.73 $\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx$

Optimal. Leaf size=18

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

Rubi [A] time = 0.30, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx &= \frac{1}{3} \text{Subst}\left(\int (b+2 c x) \left(a+b x+c x^2\right)^{13} \, dx,x,x^3\right)\\ &= \frac{1}{42} \left(a+b x^3+c x^6\right)^{14} \end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{42} x^3 \left(b+c x^3\right) \left(14 a^{13}+91 a^{12} x^3 \left(b+c x^3\right)+364 a^{11} x^6 \left(b+c x^3\right)^2+1001 a^{10} x^9 \left(b+c x^3\right)^3+2002 a^9 x^{12} \left(b+c x^3\right)^4+3003 a^8 x^{15} \left(b+c x^3\right)^5+3432 a^7 x^{18} \left(b+c x^3\right)^6+3003 a^6 x^{21} \left(b+c x^3\right)^7+2002 a^5 x^{24} \left(b+c x^3\right)^8+1001 a^4 x^{27} \left(b+c x^3\right)^9+364 a^3 x^{30} \left(b+c x^3\right)^{10}+91 a^2 x^{33} \left(b+c x^3\right)^{11}+14 a x^{36} \left(b+c x^3\right)^{12}+x^{39} \left(b+c x^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x
^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^
8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x
^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3
*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^1
2 + x^39*(b + c*x^3)^13))/42
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13, x]
```

fricas [B] time = 0.77, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 + 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 + 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 + 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 + 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 + 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 + 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 + 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 + 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 + 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 + 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 + 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
 + 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 + 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 + 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 + 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 + 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 + 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 + 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 + 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 +
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 + 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 + 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 + 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 + 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 + 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 + 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 + 572/7*x^42*c^7
*a^7 + 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 + 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 + 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 + 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 + 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 + 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 + 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 + 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 + 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 + 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 + 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 + 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 + 143/3*x^30*c^5*a^
9 + 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 + 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 + 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 + 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 + 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 + 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 + 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 + 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 + 26/3*x^18*c^3*a^11 + 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 + 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 + 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 + 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 + 1/3*x^6*c*a^13 + 1/3*x^3*b*a^13
```

giac [B] time = 0.61, size = 246, normalized size = 13.67

$$\frac{1}{42}\left(cx^6+bx^3\right)^{14}+\frac{1}{3}\left(cx^6+bx^3\right)^{13}a+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2+\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4+\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6+\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8+\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}+\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11}+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}+\frac{1}{3}\left(cx^6+bx^3\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
```

```
(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13
```

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.55, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

mupad [B] time = 3.18, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 + (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 + 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 + 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 + (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 + 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 + 1716*a^5*b^2*c^7) + x^39*((a*b^1
3)/3 + 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 + 2860*a^4*b^7*
```

```
c^3 + 6006*a^5*b^5*c^4 + 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 + 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 + 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 + 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 + (26*a^11*c^3)/3 + (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) + x^66*((26*a^3*c^11)/3 + (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 + 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 + (143*a^9*
c^5)/3 + 429*a^5*b^8*c + 2002*a^6*b^6*c^2 + 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) + x^54*((143*a^5*c^9)/3 + (143*b^10*c^4)/6 + 429*a*b^8*c^5 + 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 + (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 + 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 + 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 + (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 + 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 + x^6*((a^13*c)/3
 + (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 + (c^12*x^78*(2*a*c + 13*b^2))/6 + (143*a^7*b*x^21*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (13*a^3*b*x^3
3*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/3 + (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c + 2*b^2))/3 + (13*b*c^11*x^75*(a*c + 2*b^2
))/3
```

sympy [B] time = 0.34, size = 1394, normalized size = 77.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b
**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c*
*12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*
a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*
c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 +
1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7
 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716
*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9
/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 42
9*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3
*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*
b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3
+ 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a*
*4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**
2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b
**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2
+ 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5
 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a
**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5
005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3
*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c**
4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3
*b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
```

```
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b*
*7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 +
 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a*
*10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) +
 x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**
8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3)
+ x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a
**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
```

3.74 $\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.96

$$\frac{\left(a+x^n\left(b+cx^n\right)\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + x^n*(b + c*x^n))^14/(14*n)
```

IntegrateAlgebraic [B] time = 0.35, size = 1485, normalized size = 64.57

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(14*a^13 + 91*a^12*b*x^n + 364*a^11*b^2*x^(2*n) + 91*a^12*
c*x^(2*n) + 1001*a^10*b^3*x^(3*n) + 728*a^11*b*c*x^(3*n) + 2002*a^9*b^4*x^(
4*n) + 3003*a^10*b^2*c*x^(4*n) + 364*a^11*c^2*x^(4*n) + 3003*a^8*b^5*x^(5*n
) + 8008*a^9*b^3*c*x^(5*n) + 3003*a^10*b*c^2*x^(5*n) + 3432*a^7*b^6*x^(6*n)
 + 15015*a^8*b^4*c*x^(6*n) + 12012*a^9*b^2*c^2*x^(6*n) + 1001*a^10*c^3*x^(6
*n) + 3003*a^6*b^7*x^(7*n) + 20592*a^7*b^5*c*x^(7*n) + 30030*a^8*b^3*c^2*x^
(7*n) + 8008*a^9*b*c^3*x^(7*n) + 2002*a^5*b^8*x^(8*n) + 21021*a^6*b^6*c*x^(
8*n) + 51480*a^7*b^4*c^2*x^(8*n) + 30030*a^8*b^2*c^3*x^(8*n) + 2002*a^9*c^4
*x^(8*n) + 1001*a^4*b^9*x^(9*n) + 16016*a^5*b^7*c*x^(9*n) + 63063*a^6*b^5*c
^2*x^(9*n) + 68640*a^7*b^3*c^3*x^(9*n) + 15015*a^8*b*c^4*x^(9*n) + 364*a^3*
b^10*x^(10*n) + 9009*a^4*b^8*c*x^(10*n) + 56056*a^5*b^6*c^2*x^(10*n) + 1051
05*a^6*b^4*c^3*x^(10*n) + 51480*a^7*b^2*c^4*x^(10*n) + 3003*a^8*c^5*x^(10*n
) + 91*a^2*b^11*x^(11*n) + 3640*a^3*b^9*c*x^(11*n) + 36036*a^4*b^7*c^2*x^(1
1*n) + 112112*a^5*b^5*c^3*x^(11*n) + 105105*a^6*b^3*c^4*x^(11*n) + 20592*a^
7*b*c^5*x^(11*n) + 14*a*b^12*x^(12*n) + 1001*a^2*b^10*c*x^(12*n) + 16380*a^
3*b^8*c^2*x^(12*n) + 84084*a^4*b^6*c^3*x^(12*n) + 140140*a^5*b^4*c^4*x^(12*
n) + 63063*a^6*b^2*c^5*x^(12*n) + 3432*a^7*c^6*x^(12*n) + b^13*x^(13*n) + 1
68*a*b^11*c*x^(13*n) + 5005*a^2*b^9*c^2*x^(13*n) + 43680*a^3*b^7*c^3*x^(13*
n) + 126126*a^4*b^5*c^4*x^(13*n) + 112112*a^5*b^3*c^5*x^(13*n) + 21021*a^6*
b*c^6*x^(13*n) + 13*b^12*c*x^(14*n) + 924*a*b^10*c^2*x^(14*n) + 15015*a^2*b
^8*c^3*x^(14*n) + 76440*a^3*b^6*c^4*x^(14*n) + 126126*a^4*b^4*c^5*x^(14*n)
+ 56056*a^5*b^2*c^6*x^(14*n) + 3003*a^6*c^7*x^(14*n) + 78*b^11*c^2*x^(15*n)
 + 3080*a*b^9*c^3*x^(15*n) + 30030*a^2*b^7*c^4*x^(15*n) + 91728*a^3*b^5*c^5
*x^(15*n) + 84084*a^4*b^3*c^6*x^(15*n) + 16016*a^5*b*c^7*x^(15*n) + 286*b^1
0*c^3*x^(16*n) + 6930*a*b^8*c^4*x^(16*n) + 42042*a^2*b^6*c^5*x^(16*n) + 764
40*a^3*b^4*c^6*x^(16*n) + 36036*a^4*b^2*c^7*x^(16*n) + 2002*a^5*c^8*x^(16*n
) + 715*b^9*c^4*x^(17*n) + 11088*a*b^7*c^5*x^(17*n) + 42042*a^2*b^5*c^6*x^(
17*n) + 43680*a^3*b^3*c^7*x^(17*n) + 9009*a^4*b*c^8*x^(17*n) + 1287*b^8*c^5
*x^(18*n) + 12936*a*b^6*c^6*x^(18*n) + 30030*a^2*b^4*c^7*x^(18*n) + 16380*a
^3*b^2*c^8*x^(18*n) + 1001*a^4*c^9*x^(18*n) + 1716*b^7*c^6*x^(19*n) + 11088
*a*b^5*c^7*x^(19*n) + 15015*a^2*b^3*c^8*x^(19*n) + 3640*a^3*b*c^9*x^(19*n)
+ 1716*b^6*c^7*x^(20*n) + 6930*a*b^4*c^8*x^(20*n) + 5005*a^2*b^2*c^9*x^(20*
n) + 364*a^3*c^10*x^(20*n) + 1287*b^5*c^8*x^(21*n) + 3080*a*b^3*c^9*x^(21*n
) + 1001*a^2*b*c^10*x^(21*n) + 715*b^4*c^9*x^(22*n) + 924*a*b^2*c^10*x^(22*
n) + 91*a^2*c^11*x^(22*n) + 286*b^3*c^10*x^(23*n) + 168*a*b*c^11*x^(23*n) +
 78*b^2*c^11*x^(24*n) + 14*a*c^12*x^(24*n) + 13*b*c^12*x^(25*n) + c^13*x^(2
6*n)))/(14*n)
```

fricas [B] time = 1.22, size = 1297, normalized size = 56.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12 +
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 +
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 +
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 + 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
```

```
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^(1
4*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 2002*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*
b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*
b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b
^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(11*a^9*b^5
 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 + 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
+ 2*a^13*c)*x^(2*n))/n
```

giac [B] time = 1.00, size = 1693, normalized size = 73.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
+ 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c
^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) + 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) + 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) + 200
```

```
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) + 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) + 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) + 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) + 364*a^11*c^3*x^(6*n) + 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) +
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) + 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) + 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) + 14*a^13*c*x^(2*n) + 14*a^13*b*x^n)/n
```

maple [B] time = 0.06, size = 2042, normalized size = 88.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n)+a)^13,x)
```

```
[Out] 26*b^11*c^3/n*(x^n)^17+1716/7/n*(x^n)^14*a^7*c^7+1716/7*b^7*a^7/n*(x^n)^7+1
43*b^9*c^5/n*(x^n)^19+26*b^3*c^11/n*(x^n)^25+a*b^13/n*(x^n)^13+143*a^5*b^9/
n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+143/2*a^10/n*(x^
n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+143*b^5*a^9/n*(x^n)^5+26*b^11*a^3/n*(x^n)^
11+b^13*c/n*(x^n)^15+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+26*a^
11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+143*a^9/n*(x^n)^10*c^5+143/2*a^4/n
*(x^n)^10*b^10+429/2*c^8/n*(x^n)^22*b^6+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)
^26*b^2+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12+143*c^9/n*(x^n)^1
8*a^5+143/2*c^4/n*(x^n)^18*b^10+143/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)
^20*b^8+26*c^11/n*(x^n)^22*a^3+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12+26*a^11*b^3/n*(x^n)^3+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*
b^4+a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2+b*a^13/n*x^n+b*c^13/n*(x^n)^27
+1287*b^5*c^8/n*(x^n)^21*a+78*b*c^11/n*(x^n)^23*a^2+286*b^3*c^10/n*(x^n)^23
*a+286*b*a^10/n*(x^n)^7*c^3+1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)
^7*c+715*b*c^9/n*(x^n)^19*a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x
^n)^19*a^2+1716*b^7*c^6/n*(x^n)^19*a+5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/
n*(x^n)^12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c
^2+1/14*c^14/n*(x^n)^28+715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^14*b^14+6435/2
*a^8/n*(x^n)^10*b^2*c^4+8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6
*c^2+1287*a^5/n*(x^n)^10*b^8*c+1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n
)^8*b^4*c^2+1716*a^7/n*(x^n)^8*b^6*c+78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n
*(x^n)^5*c+1287*b*a^8/n*(x^n)^11*c^5+1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c^3+1
430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c+715*a^9*b/n*(x^n)^9*c^4
+4290*a^8*b^3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)
^9*c+286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2+8580*b^3*a^7/n*(
x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3+5148*b^7*a^5/n*(x^n)^11*c^2+715*b^
9*a^4/n*(x^n)^11*c+1716*b*c^7/n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^15*a^5+1
8018*b^5*c^5/n*(x^n)^15*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n
)^15*a^2+78*b^11*c^2/n*(x^n)^15*a+13*b*c^12/n*(x^n)^25*a+1430*c^9/n*(x^n)^2
0*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4+1716*c^7/n*(x^n)^20*a*b^6+429*c^10/
n*(x^n)^22*a^2*b^2+715*c^9/n*(x^n)^22*a*b^4+13*a^12*b/n*(x^n)^3*c+78*c^11/n
*(x^n)^24*a*b^2+78*a^11/n*(x^n)^4*b^2*c+429*a^10/n*(x^n)^6*b^2*c^2+1287*b*c
^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4+12012*b^5*c^6/n*(x^n)^17*a^3+
5148*b^7*c^5/n*(x^n)^17*a^2+715*b^9*c^4/n*(x^n)^17*a+6006/n*(x^n)^14*a^6*b^
2*c^6+18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4+4290/n*(x^n
)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2+13/n*(x^n)^14*a*b^12*c+286*a^3
/n*(x^n)^12*b^10*c+5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4
+12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2*b^8+286*c^3/n*(x^n)
^16*a*b^10+6435/2*c^8/n*(x^n)^18*a^4*b^2+8580*c^7/n*(x^n)^18*a^3*b^4+6006*c
^6/n*(x^n)^18*a^2*b^6+1287*c^5/n*(x^n)^18*a*b^8
```

maxima [B] time = 0.86, size = 2041, normalized size = 88.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n + 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n +
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n + 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n + 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n + 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n + 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n + 1716/7*a^7*c^7*x^(14*n)/n + a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n + 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n + 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n + 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n + 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n + 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n + 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n + 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n + 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n + 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n + 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n + 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n + 143*a^9*c^5*x^(10*n)/n + 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n + 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n + 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n + 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n + 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n + 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n + 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n + 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n + 26
*a^11*c^3*x^(6*n)/n + 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n + 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n + 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n + 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n + a^13*c*x^(2*n)/n + a^13*b*x^n/n
```

mupad [B] time = 5.78, size = 1395, normalized size = 60.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 + 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148
*a^5*b^2*c^7))/n + (x^(12*n + 1)*(a*b^13 + 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
```

```
5))/n + (x^(14*n + 1)*(b^13*c + 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n + (x^(21*n + 1)*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n + (x^
(17*n + 1)*(143*a^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
+ (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 + 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 + 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
+ 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
+ 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) + (a^12*x^(n + 1)*(a*c + (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n + (c^12
*x^(25*n + 1)*(a*c + (13*b^2)/2))/n + (a^13*b*x)/n + (143*a^7*b*x^(6*n + 1)
*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 +
30*a^3*b^2*c^3 + 12*a*b^6*c))/n + (13*a^3*b*x^(10*n + 1)*(2*b^10 + 99*a^5*c
^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 + 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 + 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c + 2*b^2))/n + (13*b*c^11*x^(24*
n + 1)*(a*c + 2*b^2))/n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.75 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (a - b*x - c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 11.17

$$\frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7-2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9-364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}-14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x
)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b
 + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x
^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)*(-a + b*x + c*x^2)^13, x]
```

fricas [B] time = 0.89, size = 1450, normalized size = 80.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 - x^26*c^13*a + 26*x^25*c
^11*b^3 - 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 - 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 - 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 - 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 - 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 - 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
3*a^2 - 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 - 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 - 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 - 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 - 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 - 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 - 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 - 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 - 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 - 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 - 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 - 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 - 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 - 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 - 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 -
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 - 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 - 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 - 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 - 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 - 1716/7*x^14*c^7*a^7 - x^13*b^13*a + 78*x^13*c*
b^11*a^2 - 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 - 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 - 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
- 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 - 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 - 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 - 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 - 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 - 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
- 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 - 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 - 143*x^10*c^5*a^9 - 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 - 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 - 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 - 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 - 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 - 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 - 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 - 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 - 26*x^6*c^3*a^11 - 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 - 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 - 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 - 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 -
x^2*c*a^13 - x*b*a^13
```

giac [B] time = 0.47, size = 218, normalized size = 12.11

$$\frac{1}{14}\left(cx^2+bx\right)^{14}-\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2-26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4-143\left(cx^2+bx\right)^{9}a^5+\frac{429}{2}\left(cx^2+bx\right)^{8}a^6-\frac{1716}{7}\left(cx^2+bx\right)^{7}a^7+\frac{429}{2}\left(cx^2+bx\right)^{6}a^8-143\left(cx^2+bx\right)^{5}a^9+\frac{143}{2}\left(cx^2+bx\right)^{4}a^{10}-26\left(cx^2+bx\right)^{3}a^{11}+\frac{13}{2}\left(cx^2+bx\right)^{2}a^{12}-\left(cx^2+bx\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13
```

maple [B] time = 0.00, size = 47685, normalized size = 2649.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.44, size = 16, normalized size = 0.89

$$\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x - a)^14
```

mupad [B] time = 1.38, size = 1208, normalized size = 67.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148*a^5*b^2*c^7) - x^13*(a*b^13 - 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 - 12012*a^6*b^3*c^5) + x^15*(b^13*c - 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*
b^2*c^2) - x^22*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) - x^18*(143*a^5*c^9 - (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 - (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 -
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 - 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 - x^2*(a^13*c - (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 - 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/2 + b
*c^13*x^27 - (c^12*x^26*(2*a*c - 13*b^2))/2 - a^13*b*x - (143*a^7*b*x^7*(12
*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 - 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) - 13*a^3*b*x
^11*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 - 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 -
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c) - 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 - 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 - 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c - 2*b^2) - 13*b*c^11*x^25*(a*c - 2*b^2)
```

sympy [B] time = 0.36, size = 1326, normalized size = 73.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c**
12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 - 78
*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a
```

```
**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 - 171
6*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4*b**2*c**
8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**5 + 143*b*
*10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b
**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(
429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x*
*15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580*
a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(
-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4
*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**
14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*
c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1
3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 -
12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b
**12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c
**3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*
a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c*
*2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x
**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716*
a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c*
*2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1
0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 +
 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*
c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13*
c + 13*a**12*b**2/2)
```

3.76 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=20

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

Rubi [A] time = 0.32, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\ &= \frac{1}{28}\left(a-bx^2-cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(-14a^{13}+91a^{12}x^2\left(b+cx^2\right)-364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3-2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5-3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7-2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9-364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}-14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^
8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x
^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3
*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^1
2 + x^26*(b + c*x^2)^13))/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] IntegrateAlgebraic[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13, x]
```

fricas [B] time = 0.81, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 - 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 - 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 - 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 - 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 - 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 - 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 - 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 - 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 - 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 - 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 - 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 -
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 - 1287/2
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 - 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 - 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 - 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 - 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
- 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 - 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 - 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 - 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 - 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 - 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 -
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 - 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 - 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 - 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 - 858/7*x^28*c^7*a
^7 - 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 - 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 - 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 - 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 - 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 - 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 - 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 - 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 - 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 - 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 - 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 - 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 - 143/2*x^20*c^5*a^
9 - 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 - 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 - 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 - 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 - 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 - 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 - 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 - 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 - 13*x^12*c^3*a^11 - 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
- 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 - 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 - 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 - 1/2*x^4*
c*a^13 - 1/2*x^2*b*a^13
```

giac [B] time = 0.57, size = 246, normalized size = 12.30

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}-\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2-13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4-\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6-\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8-\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}-13\left(cx^4+bx^2\right)^{3}a^{11}+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}-\frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 - 1/2*(c*x^4 + b*x^2)*a^13
```

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.52, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

mupad [B] time = 3.25, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 - 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 - 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 - 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 - 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
- 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 - 2574*a^5*b^2*c^7) - x^26*((a*b
^13)/2 - 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 - 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 - 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 - 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 - 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 - 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 - 13*a^11*c^3 - (715*a^9*b^4*c)/2
```

```
+ (429*a^10*b^2*c^2)/2) - x^44*(13*a^3*c^11 - (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 - (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 - (143*a^9*c^5)/2 -
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 - 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) - x^36*((143*a^5*c^9)/2 - (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 - 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 -
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 - 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 - 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 - 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 - 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^14*x^56)/28 - x^4*((a^13*c)/
2 - (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 - (c^12*x^52*(2*a*c - 13*b^2))/4 - (143*a^7*b*x^14*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 - (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 - (13*a^3*b*x^2
2*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/2 - (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c - 2*b^2))/2 - (13*b*c^11*x^50*(a*c - 2*b^2
))/2
```

sympy [B] time = 0.36, size = 1384, normalized size = 69.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 13
*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c
**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 - 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4
*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 + 2574*a**
2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**5*c**9/2
+ 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 - 1287*
a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*
b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 +
 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*a*b**10*c*
*3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b**3*c**6 + 9009
*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 - 39*a*b**11*c**
2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*
b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c
**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b*
*3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c**2 +
39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574*a**7*b**2*c**5
 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 - 1
43*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 - 4290*a**7*
b**3*c**4 + 6006*a**6*b**5*c**3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 -
 13*a**3*b**11) + x**20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a*
*7*b**4*c**3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b**5*c**2 +
858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 - 715*a**9*b**
2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14
```

```
*(143*a**10*b*c**3 - 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**
7/7) + x**12*(-13*a**11*c**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 +
429*a**8*b**6/4) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b*
*5/2) + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*
(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**12*b**2/4)
```

3.77 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=20

$$\frac{1}{42} \left(a-bx^3-cx^6\right)^{14}$$

Rubi [A] time = 0.31, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{1}{42} \left(a-bx^3-cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^{13} \, dx &= \frac{1}{3} \text{Subst}\left(\int (b+2cx) \left(-a+bx+cx^2\right)^{13} \, dx, x, x^3\right) \\ &= \frac{1}{42} \left(a-bx^3-cx^6\right)^{14} \end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{42} x^3 \left(b+cx^3\right) \left(-14a^{13}+91a^{12}x^3\left(b+cx^3\right)-364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3-2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5-3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7-2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9-364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}-14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a
^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*
x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^
3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^
12 + x^39*(b + c*x^3)^13))/42
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^{13} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13, x]
```

fricas [B] time = 0.80, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 - 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 - 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 - 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 - 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 - 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 - 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 - 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 - 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 - 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 - 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 - 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 - 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
 - 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 - 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 - 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 - 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 - 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 - 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 - 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 - 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 -
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 - 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 - 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 - 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 - 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 - 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 - 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 - 572/7*x^42*c^7
*a^7 - 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 - 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 - 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 - 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 - 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 - 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 - 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 - 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 - 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 - 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 - 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 - 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 - 143/3*x^30*c^5*a^
9 - 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 - 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 - 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 - 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 - 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 - 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 - 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 - 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 - 26/3*x^18*c^3*a^11 - 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 - 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 - 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 - 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 - 1/3*x^6*c*a^13 - 1/3*x^3*b*a^13
```

giac [B] time = 0.68, size = 246, normalized size = 12.30

$$\frac{1}{42}\left(cx^6+bx^3\right)^{14}-\frac{1}{3}\left(cx^6+bx^3\right)^{13}a+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2-\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4-\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6-\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8-\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}-\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11}+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}-\frac{1}{3}\left(cx^6+bx^3\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
```

```
(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13
```

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.50, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

mupad [B] time = 1.28, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 - (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 - 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 - 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 - (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 - 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 - 1716*a^5*b^2*c^7) - x^39*((a*b^1
3)/3 - 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 - 2860*a^4*b^7*
```

```
c^3 + 6006*a^5*b^5*c^4 - 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 - 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 - 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 - 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 - (26*a^11*c^3)/3 - (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) - x^66*((26*a^3*c^11)/3 - (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 - 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 - (143*a^9*
c^5)/3 - 429*a^5*b^8*c + 2002*a^6*b^6*c^2 - 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) - x^54*((143*a^5*c^9)/3 - (143*b^10*c^4)/6 + 429*a*b^8*c^5 - 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 - (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 - 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 - 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 - (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 - 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 - x^6*((a^13*c)/3
 - (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 - (c^12*x^78*(2*a*c - 13*b^2))/6 - (143*a^7*b*x^21*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 - (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 - (13*a^3*b*x^3
3*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/3 - (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c - 2*b^2))/3 - (13*b*c^11*x^75*(a*c - 2*b^2
))/3
```

sympy [B] time = 0.36, size = 1394, normalized size = 69.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 13
*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2
*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 - 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a**2*b
**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(14
3*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**3*c**8 +
 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(-143*a**
5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**
6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a*
*4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/
3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 286*a*b**10*
c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a**5*b**3*c**6 +
6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 - 26*a*b*
*11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 600
6*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**
10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*
b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9*c**2
/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 - 1716*a**7*b**2
*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 -
 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 - 2860*a**7
*b**3*c**4 + 4004*a**6*b**5*c**3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3
- 26*a**3*b**11/3) + x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860
*a**7*b**4*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
```

```
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b**5*c**2 +
572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 - 1430*a**9*b*
*2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*b**6*c + 143*a**6*b**8/2) + x*
*21*(286*a**10*b*c**3/3 - 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**
7*b**7/7) + x**18*(-26*a**11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c
/3 + 143*a**8*b**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*
a**9*b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b**4/6)
 + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/3 + 13*a**12*b*
*2/6)
```

3.78 $\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{\left(x^n\left(b+cx^n\right)-a\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^14/(14*n)
```

IntegrateAlgebraic [B] time = 0.38, size = 1485, normalized size = 59.40

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(-14*a^13 + 91*a^12*b*x^n - 364*a^11*b^2*x^(2*n) + 91*a^12
*c*x^(2*n) + 1001*a^10*b^3*x^(3*n) - 728*a^11*b*c*x^(3*n) - 2002*a^9*b^4*x^
(4*n) + 3003*a^10*b^2*c*x^(4*n) - 364*a^11*c^2*x^(4*n) + 3003*a^8*b^5*x^(5*
n) - 8008*a^9*b^3*c*x^(5*n) + 3003*a^10*b*c^2*x^(5*n) - 3432*a^7*b^6*x^(6*n
) + 15015*a^8*b^4*c*x^(6*n) - 12012*a^9*b^2*c^2*x^(6*n) + 1001*a^10*c^3*x^(
6*n) + 3003*a^6*b^7*x^(7*n) - 20592*a^7*b^5*c*x^(7*n) + 30030*a^8*b^3*c^2*x
^(7*n) - 8008*a^9*b*c^3*x^(7*n) - 2002*a^5*b^8*x^(8*n) + 21021*a^6*b^6*c*x^
(8*n) - 51480*a^7*b^4*c^2*x^(8*n) + 30030*a^8*b^2*c^3*x^(8*n) - 2002*a^9*c^
4*x^(8*n) + 1001*a^4*b^9*x^(9*n) - 16016*a^5*b^7*c*x^(9*n) + 63063*a^6*b^5*
c^2*x^(9*n) - 68640*a^7*b^3*c^3*x^(9*n) + 15015*a^8*b*c^4*x^(9*n) - 364*a^3
*b^10*x^(10*n) + 9009*a^4*b^8*c*x^(10*n) - 56056*a^5*b^6*c^2*x^(10*n) + 105
105*a^6*b^4*c^3*x^(10*n) - 51480*a^7*b^2*c^4*x^(10*n) + 3003*a^8*c^5*x^(10*
n) + 91*a^2*b^11*x^(11*n) - 3640*a^3*b^9*c*x^(11*n) + 36036*a^4*b^7*c^2*x^(
11*n) - 112112*a^5*b^5*c^3*x^(11*n) + 105105*a^6*b^3*c^4*x^(11*n) - 20592*a
^7*b*c^5*x^(11*n) - 14*a*b^12*x^(12*n) + 1001*a^2*b^10*c*x^(12*n) - 16380*a
^3*b^8*c^2*x^(12*n) + 84084*a^4*b^6*c^3*x^(12*n) - 140140*a^5*b^4*c^4*x^(12
*n) + 63063*a^6*b^2*c^5*x^(12*n) - 3432*a^7*c^6*x^(12*n) + b^13*x^(13*n) -
168*a*b^11*c*x^(13*n) + 5005*a^2*b^9*c^2*x^(13*n) - 43680*a^3*b^7*c^3*x^(13
*n) + 126126*a^4*b^5*c^4*x^(13*n) - 112112*a^5*b^3*c^5*x^(13*n) + 21021*a^6
*b*c^6*x^(13*n) + 13*b^12*c*x^(14*n) - 924*a*b^10*c^2*x^(14*n) + 15015*a^2*
b^8*c^3*x^(14*n) - 76440*a^3*b^6*c^4*x^(14*n) + 126126*a^4*b^4*c^5*x^(14*n)
 - 56056*a^5*b^2*c^6*x^(14*n) + 3003*a^6*c^7*x^(14*n) + 78*b^11*c^2*x^(15*n
) - 3080*a*b^9*c^3*x^(15*n) + 30030*a^2*b^7*c^4*x^(15*n) - 91728*a^3*b^5*c^
5*x^(15*n) + 84084*a^4*b^3*c^6*x^(15*n) - 16016*a^5*b*c^7*x^(15*n) + 286*b^
10*c^3*x^(16*n) - 6930*a*b^8*c^4*x^(16*n) + 42042*a^2*b^6*c^5*x^(16*n) - 76
440*a^3*b^4*c^6*x^(16*n) + 36036*a^4*b^2*c^7*x^(16*n) - 2002*a^5*c^8*x^(16*
n) + 715*b^9*c^4*x^(17*n) - 11088*a*b^7*c^5*x^(17*n) + 42042*a^2*b^5*c^6*x^
(17*n) - 43680*a^3*b^3*c^7*x^(17*n) + 9009*a^4*b*c^8*x^(17*n) + 1287*b^8*c^
5*x^(18*n) - 12936*a*b^6*c^6*x^(18*n) + 30030*a^2*b^4*c^7*x^(18*n) - 16380*
a^3*b^2*c^8*x^(18*n) + 1001*a^4*c^9*x^(18*n) + 1716*b^7*c^6*x^(19*n) - 1108
8*a*b^5*c^7*x^(19*n) + 15015*a^2*b^3*c^8*x^(19*n) - 3640*a^3*b*c^9*x^(19*n)
 + 1716*b^6*c^7*x^(20*n) - 6930*a*b^4*c^8*x^(20*n) + 5005*a^2*b^2*c^9*x^(20
*n) - 364*a^3*c^10*x^(20*n) + 1287*b^5*c^8*x^(21*n) - 3080*a*b^3*c^9*x^(21*
n) + 1001*a^2*b*c^10*x^(21*n) + 715*b^4*c^9*x^(22*n) - 924*a*b^2*c^10*x^(22
*n) + 91*a^2*c^11*x^(22*n) + 286*b^3*c^10*x^(23*n) - 168*a*b*c^11*x^(23*n)
+ 78*b^2*c^11*x^(24*n) - 14*a*c^12*x^(24*n) + 13*b*c^12*x^(25*n) + c^13*x^(
26*n)))/(14*n)
```

fricas [B] time = 0.86, size = 1299, normalized size = 51.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 -
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 -
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 -
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
```

```
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(1
4*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*
b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*
b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b
^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(11*a^9*b^5
 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 - 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
- 2*a^13*c)*x^(2*n))/n
```

giac [B] time = 1.09, size = 1693, normalized size = 67.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
- 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c
^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) - 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) - 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) - 200
```

```
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) - 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) - 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) - 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) - 364*a^11*c^3*x^(6*n) - 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) -
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) - 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) - 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) - 14*a^13*c*x^(2*n) - 14*a^13*b*x^n)/n
```

maple [B] time = 0.06, size = 2046, normalized size = 81.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 26*b^11*c^3/n*(x^n)^17-1716/7/n*(x^n)^14*a^7*c^7-1716/7*b^7*a^7/n*(x^n)^7+1
43*b^9*c^5/n*(x^n)^19+26*b^3*c^11/n*(x^n)^25-a*b^13/n*(x^n)^13-143*a^5*b^9/
n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+143/2*a^10/n*(x^
n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8-143*b^5*a^9/n*(x^n)^5-26*b^11*a^3/n*(x^n)^
11+b^13*c/n*(x^n)^15+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4-26*a^
11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6-143*a^9/n*(x^n)^10*c^5+143/2*a^4/n
*(x^n)^10*b^10+429/2*c^8/n*(x^n)^22*b^6-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)
^26*b^2+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12-143*c^9/n*(x^n)^1
8*a^5+143/2*c^4/n*(x^n)^18*b^10+143/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)
^20*b^8-26*c^11/n*(x^n)^22*a^3+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12-26*a^11*b^3/n*(x^n)^3+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*
b^4-a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2-a^13*b/n*x^n+b*c^13/n*(x^n)^27
-1287*b^5*c^8/n*(x^n)^21*a+78*b*c^11/n*(x^n)^23*a^2-286*b^3*c^10/n*(x^n)^23
*a+286*b*a^10/n*(x^n)^7*c^3-1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)
^7*c+715*b*c^9/n*(x^n)^19*a^4-4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x
^n)^19*a^2-1716*b^7*c^6/n*(x^n)^19*a-5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/
n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c
^2+1/14*c^14/n*(x^n)^28-715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^14*b^14+6435/2
*a^8/n*(x^n)^10*b^2*c^4-8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6
*c^2-1287*a^5/n*(x^n)^10*b^8*c-1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n
)^8*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c-78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n
*(x^n)^5*c+1287*b*a^8/n*(x^n)^11*c^5-1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c^3-1
430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c-715*a^9*b/n*(x^n)^9*c^4
+4290*a^8*b^3/n*(x^n)^9*c^3-5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)
^9*c-286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2-8580*b^3*a^7/n*(
x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3-5148*b^7*a^5/n*(x^n)^11*c^2+715*b^
9*a^4/n*(x^n)^11*c+1716*b*c^7/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^15*a^5+1
8018*b^5*c^5/n*(x^n)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n
)^15*a^2-78*b^11*c^2/n*(x^n)^15*a-13*b*c^12/n*(x^n)^25*a-1430*c^9/n*(x^n)^2
0*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4-1716*c^7/n*(x^n)^20*a*b^6+429*c^10/
n*(x^n)^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^4+13*a^12*b/n*(x^n)^3*c-78*c^11/n
*(x^n)^24*a*b^2-78*a^11/n*(x^n)^4*b^2*c+429*a^10/n*(x^n)^6*b^2*c^2-1287*b*c
^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4-12012*b^5*c^6/n*(x^n)^17*a^3+
5148*b^7*c^5/n*(x^n)^17*a^2-715*b^9*c^4/n*(x^n)^17*a+6006/n*(x^n)^14*a^6*b^
2*c^6-18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4-4290/n*(x^n
)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2-13/n*(x^n)^14*a*b^12*c-286*a^3
/n*(x^n)^12*b^10*c-5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4
-12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2*b^8-286*c^3/n*(x^n)
^16*a*b^10+6435/2*c^8/n*(x^n)^18*a^4*b^2-8580*c^7/n*(x^n)^18*a^3*b^4+6006*c
^6/n*(x^n)^18*a^2*b^6-1287*c^5/n*(x^n)^18*a*b^8
```

maxima [B] time = 0.83, size = 2045, normalized size = 81.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n - 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n -
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n - 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n - 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n - 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n - 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n - 1716/7*a^7*c^7*x^(14*n)/n - a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n - 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n - 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n - 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n - 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n - 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n - 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n - 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n - 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n - 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n - 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n - 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n - 143*a^9*c^5*x^(10*n)/n - 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n - 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n - 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n - 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n - 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n - 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n - 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n - 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n - 26
*a^11*c^3*x^(6*n)/n - 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n - 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n - 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n - 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n - a^13*c*x^(2*n)/n - a^13*b*x^n/n
```

mupad [B] time = 5.78, size = 1401, normalized size = 56.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148
*a^5*b^2*c^7))/n - (x^(12*n + 1)*(a*b^13 - 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
```

```
5))/n + (x^(14*n + 1)*(b^13*c - 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n - (x^(21*n + 1)*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n - (x^
(17*n + 1)*(143*a^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
- (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 - 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 - 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
- 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
- 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) - (a^12*x^(n + 1)*(a*c - (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n - (c^12
*x^(25*n + 1)*(a*c - (13*b^2)/2))/n - (a^13*b*x)/n - (143*a^7*b*x^(6*n + 1)
*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) - (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 -
30*a^3*b^2*c^3 - 12*a*b^6*c))/n - (13*a^3*b*x^(10*n + 1)*(2*b^10 - 99*a^5*c
^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n - (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 - 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 - 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c - 2*b^2))/n - (13*b*c^11*x^(24*
n + 1)*(a*c - 2*b^2))/n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.79 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13, x]
```

fricas [B] time = 0.55, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8+143x^{19}c^5b^9+\frac{143}{2}x^{18}c^4b^{10}+26x^{17}c^3b^{11}+\frac{13}{2}x^{16}c^2b^{12}+x^{15}cb^{13}+\frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

giac [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28}+b\,c^{13}x^{27}+\frac{13}{2}b^{2}c^{12}x^{26}+26b^{3}c^{11}x^{25}+\frac{143}{2}b^{4}c^{10}x^{24}+143b^{5}c^{9}x^{23}+\frac{429}{2}b^{6}c^{8}x^{22}+\frac{1716}{7}b^{7}c^{7}x^{21}+\frac{429}{2}b^{8}c^{6}x^{20}+143b^{9}c^{5}x^{19}+\frac{143}{2}b^{10}c^{4}x^{18}+26b^{11}c^{3}x^{17}+\frac{13}{2}b^{12}c^{2}x^{16}+b^{13}c\,x^{15}+\frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

mupad [B] time = 2.09, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^{2}x^{16}}{2}+26b^{11}c^{3}x^{17}+\frac{143b^{10}c^{4}x^{18}}{2}+143b^{9}c^{5}x^{19}+\frac{429b^{8}c^{6}x^{20}}{2}+\frac{1716b^{7}c^{7}x^{21}}{7}+\frac{429b^{6}c^{8}x^{22}}{2}+143b^{5}c^{9}x^{23}+\frac{143b^{4}c^{10}x^{24}}{2}+26b^{3}c^{11}x^{25}+\frac{13b^{2}c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

3.80 $\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{1584, 446, 74\}$

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx &= \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^2\right)\\
&= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
```

```
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] IntegrateAlgebraic[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13, x]
```

fricas [B] time = 0.72, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^{2}+13x^{50}c^{11}b^{3}+\frac{143}{4}x^{48}c^{10}b^{4}+\frac{143}{2}x^{46}c^{9}b^{5}+\frac{429}{4}x^{44}c^{8}b^{6}+\frac{858}{7}x^{42}c^{7}b^{7}+\frac{429}{4}x^{40}c^{6}b^{8}+\frac{143}{2}x^{38}c^{5}b^{9}+\frac{143}{4}x^{36}c^{4}b^{10}+13x^{34}c^{3}b^{11}+\frac{13}{4}x^{32}c^{2}b^{12}+\frac{1}{2}x^{30}cb^{13}+\frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

giac [A] time = 0.38, size = 15, normalized size = 0.94

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{143}{4}b^{10}c^{4}x^{36}+13b^{11}c^{3}x^{34}+\frac{13}{4}b^{12}c^{2}x^{32}+\frac{1}{2}b^{13}c\,x^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

maxima [B] time = 0.43, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{143}{4}b^{10}c^{4}x^{36}+13\,b^{11}c^{3}x^{34}+\frac{13}{4}b^{12}c^{2}x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

sympy [B] time = 0.13, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

3.81 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = $\{1584, 446, 74\}$

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx &= \int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx\\
&= \frac{1}{3}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)\\
&= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
```

```
(572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(b + 2cx^3\right) \left(bx^3 + cx^6\right)^{13} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13, x]
```

fricas [B] time = 0.74, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6b^8 + \frac{143}{3}x^{57}c^5b^9 + \frac{143}{6}x^{54}c^4b^{10} + \frac{26}{3}x^{51}c^3b^{11} + \frac{13}{6}x^{48}c^2b^{12} + \frac{1}{3}x^{45}cb^{13} + \frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

giac [A] time = 0.51, size = 15, normalized size = 0.94

$$\frac{1}{42} \left(cx^6 + bx^3\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}b\,c^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}c\,x^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

maxima [B] time = 0.44, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

sympy [B] time = 0.13, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

3.82 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}dx &= \int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx\\ &= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}\\ &= \frac{x^{14n}(b+cx^n)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

IntegrateAlgebraic [A] time = 0.06, size = 21, normalized size = 1.00

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

fricas [B] time = 0.87, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n}+14\,bc^{13}x^{27n}+91\,b^2c^{12}x^{26n}+364\,b^3c^{11}x^{25n}+1001\,b^4c^{10}x^{24n}+2002\,b^5c^9x^{23n}+3003\,b^6c^8x^{22n}+3432\,b^7c^7x^{21n}+3003\,b^8c^6x^{20n}+2002\,b^9c^5x^{19n}+1001\,b^{10}c^4x^{18n}+364\,b^{11}c^3x^{17n}+91\,b^{12}c^2x^{16n}+14\,b^{13}cx^{15n}+b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

giac [B] time = 0.43, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n}+14\,bc^{13}x^{27n}+91\,b^2c^{12}x^{26n}+364\,b^3c^{11}x^{25n}+1001\,b^4c^{10}x^{24n}+2002\,b^5c^9x^{23n}+3003\,b^6c^8x^{22n}+3432\,b^7c^7x^{21n}+3003\,b^8c^6x^{20n}+2002\,b^9c^5x^{19n}+1001\,b^{10}c^4x^{18n}+364\,b^{11}c^3x^{17n}+91\,b^{12}c^2x^{16n}+14\,b^{13}cx^{15n}+b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

maple [B] time = 0.04, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n}+\frac{b^{13}c\,x^{15n}}{n}+\frac{13b^{12}c^2x^{16n}}{2n}+\frac{26b^{11}c^3x^{17n}}{n}+\frac{143b^{10}c^4x^{18n}}{2n}+\frac{143b^9c^5x^{19n}}{n}+\frac{429b^8c^6x^{20n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{429b^6c^8x^{22n}}{2n}+\frac{143b^5c^9x^{23n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{26b^3c^{11}x^{25n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{c^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*b^3*c^11
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

maxima [B] time = 0.48, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14\,n}+\frac{bc^{13}x^{27n}}{n}+\frac{13\,b^2c^{12}x^{26n}}{2\,n}+\frac{26\,b^3c^{11}x^{25n}}{n}+\frac{143\,b^4c^{10}x^{24n}}{2\,n}+\frac{143\,b^5c^9x^{23n}}{n}+\frac{429\,b^6c^8x^{22n}}{2\,n}+\frac{1716\,b^7c^7x^{21n}}{7\,n}+\frac{429\,b^8c^6x^{20n}}{2\,n}+\frac{143\,b^9c^5x^{19n}}{n}+\frac{143\,b^{10}c^4x^{18n}}{2\,n}+\frac{26\,b^{11}c^3x^{17n}}{n}+\frac{13\,b^{12}c^2x^{16n}}{2\,n}+\frac{b^{13}cx^{15n}}{n}+\frac{b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

mupad [B] time = 2.63, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

3.83 $\int \frac{b+2cx}{a+bx+cx^2}\,dx$

Optimal. Leaf size=11

$$\log\left(a+bx+cx^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {628}

$$\log\left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]
[Out] Log[a + b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx=\log\left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.91

$$\log(a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]
[Out] Log[a + x*(b + c*x)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(a + b*x + c*x^2),x]
[Out] IntegrateAlgebraic[(b + 2*c*x)/(a + b*x + c*x^2), x]
```

fricas [A] time = 1.06, size = 11, normalized size = 1.00

$$\log\left(cx^2+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] log(c*x^2 + b*x + a)
```

giac [A] time = 0.35, size = 12, normalized size = 1.09

$$\log\left(\left|cx^2 + bx + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x + a))
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\ln\left(c\,x^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a),x)
[Out] ln(c*x^2+b*x+a)
```

maxima [A] time = 0.45, size = 11, normalized size = 1.00

$$\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x + a)
```

mupad [B] time = 1.96, size = 11, normalized size = 1.00

$$\ln\left(c\,x^2 + b\,x + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2),x)
[Out] log(a + b*x + c*x^2)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.91

$$\log\left(a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
[Out] log(a + b*x + c*x**2)
```

3.84 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a+bx^2+cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(b+2cx^2\right)}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [A] time = 1.04, size = 15, normalized size = 0.88

$$\frac{1}{2}\log\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

giac [A] time = 1.73, size = 16, normalized size = 0.94

$$\frac{1}{2}\log\left(\left|cx^4 + bx^2 + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2 + a))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln\left(c\,x^4 + b\,x^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x)
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.88

$$\frac{1}{2}\log\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

mupad [B] time = 1.96, size = 15, normalized size = 0.88

$$\frac{\ln\left(c\,x^4 + b\,x^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x)
[Out] log(a + b*x^2 + c*x^4)/2
```

sympy [A] time = 0.28, size = 14, normalized size = 0.82

$$\frac{\log\left(a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
[Out] log(a + b*x**2 + c*x**4)/2
```

3.85 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6} dx$

Optimal. Leaf size=17

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\log\left(a+bx^3+cx^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
[Out] Log[a + b*x^3 + c*x^6]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b+2cx^3\right)}{a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.81, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6 + bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

giac [A] time = 1.09, size = 16, normalized size = 0.94

$$\frac{1}{3}\log\left(\left|cx^6 + bx^3 + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 + a))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln\left(c\,x^6 + b\,x^3 + a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x)
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6 + bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

mupad [B] time = 0.05, size = 15, normalized size = 0.88

$$\frac{\ln\left(c\,x^6 + b\,x^3 + a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)
[Out] log(a + b*x^3 + c*x^6)/3
```

sympy [A] time = 0.41, size = 14, normalized size = 0.82

$$\frac{\log\left(a + bx^3 + cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
[Out] log(a + b*x**3 + c*x**6)/3
```

3.86 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 628}

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\log\left(a+bx^n+cx^{2n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 19, normalized size = 1.00

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

IntegrateAlgebraic [A] time = 0.05, size = 19, normalized size = 1.00

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

fricas [A] time = 1.13, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^n + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

giac [A] time = 0.46, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^n + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

maple [A] time = 0.02, size = 24, normalized size = 1.26

$$\frac{\ln\left(b\,\mathrm{e}^{n\ln(x)} + c\,\mathrm{e}^{2n\ln(x)} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)+a),x)
[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

maxima [A] time = 0.60, size = 23, normalized size = 1.21

$$\frac{\log\left(\frac{cx^{2n}+bx^n+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

mupad [B] time = 2.32, size = 121, normalized size = 6.37

$$-\frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)-\ln\left(a+b\,x^n+c\,x^{2\,n}\right)\,\sqrt{4\,a\,c-b^2}}{n\,\sqrt{4\,a\,c-b^2}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2-4\,a\,c}}\right)}{n\,\sqrt{b^2-4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x)
[Out] - (2*b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^n)/(4*a*c - b^2)^(1/2)) - log(a
+ b*x^n + c*x^(2*n))*(4*a*c - b^2)^(1/2))/(n*(4*a*c - b^2)^(1/2)) - (2*b*at
anh((b + 2*c*x^n)/(b^2 - 4*a*c)^(1/2)))/(n*(b^2 - 4*a*c)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{b+2cx}{(a+bx+cx^2)^8} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(a + b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8} \, dx = -\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(a + x*(b + c*x))^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(a + b*x + c*x^2)^8, x]
```

fricas [B] time = 0.66, size = 350, normalized size = 21.88

$$-\frac{1}{7\left(c^7x^{14}+7bc^6x^{13}+7\left(3b^2c^5+ac^6\right)x^{12}+7\left(5b^3c^4+6abc^5\right)x^{11}+7\left(5b^4c^3+15ab^2c^4+3a^2c^5\right)x^{10}+7\left(3b^5c^2+20ab^3c^3+15a^2bc^4\right)x^9+7\left(b^6c+15ab^4c^2+30a^2b^2c^3+5a^3c^4\right)x^8+7a^6bx+\left(b^7+42ab^5c+210a^2b^3c^2+140a^3bc^3\right)x^7+a^7+7\left(ab^6+15a^2b^4c+30a^3b^2c^2+5a^4c^3\right)x^6+7\left(3a^2b^5+20a^3b^3c+15a^4bc^2\right)x^5+7\left(5a^3b^4+15a^4b^2c+3a^5c^2\right)x^4+7\left(5a^4b^3+6a^5bc\right)x^3+7\left(3a^5b^2+a^6c\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 +
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2
+ 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^
4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^
5*b^2 + a^6*c)*x^2)
```

giac [A] time = 0.40, size = 14, normalized size = 0.88

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{7\left(c\,x^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

mupad [B] time = 3.62, size = 358, normalized size = 22.38

$$-\frac{1}{7\left(x^5\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\left(140\,a^3\,b\,c^3+210\,a^2\,b^3\,c^2+42\,a\,b^5\,c+b^7\right)+x^3\left(42\,c\,a^5\,b+35\,a^4\,b^3\right)+x^{11}\left(35\,b^3\,c^4+42\,a\,b\,c^5\right)+x^4\left(21\,a^5\,c^2+105\,a^4\,b^2\,c+35\,a^3\,b^4\right)+x^{10}\left(21\,a^2\,c^5+105\,a\,b^2\,c^4+35\,b^4\,c^3\right)+a^7+x^6\left(35\,a^4\,c^3+210\,a^3\,b^2\,c^2+105\,a^2\,b^4\,c+7\,a\,b^6\right)+x^8\left(35\,a^3\,c^4+210\,a^2\,b^2\,c^3+105\,a\,b^4\,c^2+7\,b^6\,c\right)+c^7\,x^{14}+x^2\left(7\,c\,a^6+21\,a^5\,b^2\right)+x^{12}\left(21\,b^2\,c^5+7\,a\,c^6\right)+7\,b\,c^6\,x^{13}+7\,a^6\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 +
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*a*b
*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 + 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

sympy [**B**]   time = 4.79, size = 359, normalized size = 22.44

$$-\frac{1}{7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\left(49ac^6 + 147b^2c^5\right) + x^{11}\left(294abc^5 + 245b^3c^4\right) + x^{10}\left(147a^2c^5 + 735ab^2c^4 + 245b^4c^3\right) + x^9\left(735a^2bc^4 + 980ab^3c^3 + 147b^5c^2\right) + x^8\left(245a^3c^4 + 1470a^2b^2c^3 + 735ab^4c^2 + 49b^6c\right) + x^7\left(980a^3bc^3 + 1470a^2b^3c^2 + 294ab^5c + 7b^7\right) + x^6\left(245a^4c^3 + 1470a^3b^2c^2 + 735a^2b^4c + 49ab^6\right) + x^5\left(735a^4bc^2 + 980a^3b^3c + 147a^2b^5\right) + x^4\left(147a^5c^2 + 735a^4b^2c + 245a^3b^4\right) + x^3\left(294a^5bc + 245a^4b^3\right) + x^2\left(49a^6c + 147a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

```
[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**
6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2
*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b*
*3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*
a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 29
4*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2
*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b
**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a
**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

3.88 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$

Optimal. Leaf size=18

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 629}

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8} dx, x, x^2\right)\\ &= -\frac{1}{14(a+bx^2+cx^4)^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]

[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8, x]

fricas [B] time = 1.05, size = 352, normalized size = 19.56

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{24}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{22}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{20}+7\left(3\,b^{5}c^{2}+20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c+15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}+5\,a^{3}c^{4}\right)x^{16}+\left(b^{7}+42\,ab^{5}c+210\,a^{2}b^{3}c^{2}+140\,a^{3}bc^{3}\right)x^{14}+7\left(ab^{6}+15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}+5\,a^{4}c^{3}\right)x^{12}+7\left(3\,a^{2}b^{5}+20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{10}+7\,a^{6}bx^{2}+7\left(5\,a^{3}b^{4}+15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{8}+a^{7}+7\left(5\,a^{4}b^{3}+6\,a^{5}bc\right)x^{6}+7\left(3\,a^{5}b^{2}+a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)

giac [A] time = 6.78, size = 16, normalized size = 0.89

$$-\frac{1}{14\left(cx^{4}+bx^{2}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2 + a)^7

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x)

[Out] -1/14/(c*x^4+b*x^2+a)^7

maxima [B] time = 0.95, size = 352, normalized size = 19.56

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{24}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{22}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{20}+7\left(3\,b^{5}c^{2}+20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c+15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}+5\,a^{3}c^{4}\right)x^{16}+\left(b^{7}+42\,ab^{5}c+210\,a^{2}b^{3}c^{2}+140\,a^{3}bc^{3}\right)x^{14}+7\left(ab^{6}+15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}+5\,a^{4}c^{3}\right)x^{12}+7\left(3\,a^{2}b^{5}+20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{10}+7\,a^{6}bx^{2}+7\left(5\,a^{3}b^{4}+15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{8}+a^{7}+7\left(5\,a^{4}b^{3}+6\,a^{5}bc\right)x^{6}+7\left(3\,a^{5}b^{2}+a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)

mupad [B] time = 12.16, size = 360, normalized size = 20.00

$$-\frac{1}{14\left(x^{10}\left(105a^4bc^2+140a^3b^3c+21a^2b^5\right)+x^{18}\left(105a^2bc^4+140ab^3c^3+21b^5c^2\right)+x^{14}\left(140a^3bc^3+210a^2b^3c^2+42ab^5c+b^7\right)+x^6\left(42ca^5b+35a^4b^3\right)+x^{22}\left(35b^3c^4+42abc^5\right)+x^8\left(21a^5c^2+105a^4b^2c+35a^3b^4\right)+x^{20}\left(21a^2c^5+105ab^2c^4+35b^4c^3\right)+a^7+x^{12}\left(35a^4c^3+210a^3b^2c^2+105a^2b^4c+7ab^6\right)+x^{16}\left(35a^3c^4+210a^2b^2c^3+105ab^4c^2+7b^6c\right)+c^7x^{28}+x^4\left(7ca^6+21a^5b^2\right)+x^{24}\left(21b^2c^5+7ac^6\right)+7a^6bx^2+7bc^6x^{26}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b*c) + x^22*(35*b^3*c^4 + 42
*a*b*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^12*(7*a*b^6 + 35*a^4*c^3 + 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c + 21*a^5*b^2) + x^24*(7*a*c^6 + 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

sympy [B] time = 7.66, size = 360, normalized size = 20.00

$$-\frac{1}{14a^7+98a^6bx^2+98bc^6x^{26}+14c^7x^{28}+x^{24}\left(98ac^6+294b^2c^5\right)+x^{22}\left(588abc^5+490b^3c^4\right)+x^{20}\left(294a^2c^5+1470ab^2c^4+490b^4c^3\right)+x^{18}\left(1470a^2bc^4+1960ab^3c^3+294b^5c^2\right)+x^{16}\left(490a^3c^4+2940a^2b^2c^3+1470ab^4c^2+98b^6c\right)+x^{14}\left(1960a^3bc^3+2940a^2b^3c^2+588ab^5c+14b^7\right)+x^{12}\left(490a^4c^3+2940a^3b^2c^2+1470a^2b^4c+98ab^6\right)+x^{10}\left(1470a^4bc^2+1960a^3b^3c+294a^2b^5\right)+x^8\left(294a^5c^2+1470a^4b^2c+490a^3b^4\right)+x^6\left(588a^5bc+490a^4b^3\right)+x^4\left(98a^6c+294a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] -1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98*
a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(294
*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 +
1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b**2*c
**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a**2*b
**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3*b**2*
c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 1960*a**3*
b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c + 490*a**3
*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 294*a**5*b
**2))
```

3.89 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8} dx$

Optimal. Leaf size=18

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 629}

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{\left(a+bx^3+cx^6\right)^8} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8} dx, x, x^3\right)\\ &= -\frac{1}{21\left(a+bx^3+cx^6\right)^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(a + b*x^3 + c*x^6)^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b+2cx^3\right)}{\left(a+bx^3+cx^6\right)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8, x]
```

fricas [B] time = 1.01, size = 352, normalized size = 19.56

$$-\frac{1}{21\left(c^{7}x^{42}+7bc^{6}x^{39}+7\left(3b^{2}c^{5}+ac^{6}\right)x^{36}+7\left(5b^{3}c^{4}+6abc^{5}\right)x^{33}+7\left(5b^{4}c^{3}+15ab^{2}c^{4}+3a^{2}c^{5}\right)x^{30}+7\left(3b^{5}c^{2}+20ab^{3}c^{3}+15a^{2}bc^{4}\right)x^{27}+7\left(b^{6}c+15ab^{4}c^{2}+30a^{2}b^{2}c^{3}+5a^{3}c^{4}\right)x^{24}+\left(b^{7}+42ab^{5}c+210a^{2}b^{3}c^{2}+140a^{3}bc^{3}\right)x^{21}+7\left(ab^{6}+15a^{2}b^{4}c+30a^{3}b^{2}c^{2}+5a^{4}c^{3}\right)x^{18}+7\left(3a^{2}b^{5}+20a^{3}b^{3}c+15a^{4}bc^{2}\right)x^{15}+7\left(5a^{3}b^{4}+15a^{4}b^{2}c+3a^{5}c^{2}\right)x^{12}+7a^{6}bx^{3}+7\left(5a^{4}b^{3}+6a^{5}bc\right)x^{9}+a^{7}+7\left(3a^{5}b^{2}+a^{6}c\right)x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

giac [A] time = 22.37, size = 16, normalized size = 0.89

$$-\frac{1}{21\left(cx^{6}+bx^{3}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 + a)^7
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

maxima [B] time = 0.95, size = 352, normalized size = 19.56

$$-\frac{1}{21\left(c^{7}x^{42}+7bc^{6}x^{39}+7\left(3b^{2}c^{5}+ac^{6}\right)x^{36}+7\left(5b^{3}c^{4}+6abc^{5}\right)x^{33}+7\left(5b^{4}c^{3}+15ab^{2}c^{4}+3a^{2}c^{5}\right)x^{30}+7\left(3b^{5}c^{2}+20ab^{3}c^{3}+15a^{2}bc^{4}\right)x^{27}+7\left(b^{6}c+15ab^{4}c^{2}+30a^{2}b^{2}c^{3}+5a^{3}c^{4}\right)x^{24}+\left(b^{7}+42ab^{5}c+210a^{2}b^{3}c^{2}+140a^{3}bc^{3}\right)x^{21}+7\left(ab^{6}+15a^{2}b^{4}c+30a^{3}b^{2}c^{2}+5a^{4}c^{3}\right)x^{18}+7\left(3a^{2}b^{5}+20a^{3}b^{3}c+15a^{4}bc^{2}\right)x^{15}+7\left(5a^{3}b^{4}+15a^{4}b^{2}c+3a^{5}c^{2}\right)x^{12}+7a^{6}bx^{3}+7\left(5a^{4}b^{3}+6a^{5}bc\right)x^{9}+a^{7}+7\left(3a^{5}b^{2}+a^{6}c\right)x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

mupad [B] time = 18.21, size = 360, normalized size = 20.00

$$-\frac{1}{21\left(x^{15}\left(105a^4bc^2+140a^3b^3c+21a^2b^5\right)+x^{27}\left(105a^2bc^4+140ab^3c^3+21b^5c^2\right)+x^{21}\left(140a^3bc^3+210a^2b^3c^2+42ab^5c+b^7\right)+x^9\left(42ca^5b+35a^4b^3\right)+x^{33}\left(35b^3c^4+42abc^5\right)+x^{12}\left(21a^5c^2+105a^4b^2c+35a^3b^4\right)+x^{30}\left(21a^2c^5+105ab^2c^4+35b^4c^3\right)+a^7+x^{18}\left(35a^4c^3+210a^3b^2c^2+105a^2b^4c+7ab^6\right)+x^{24}\left(35a^3c^4+210a^2b^2c^3+105ab^4c^2+7b^6c\right)+c^7x^{42}+x^6\left(7ca^6+21a^5b^2\right)+x^{36}\left(21b^2c^5+7ac^6\right)+7a^6bx^3+7bc^6x^{39}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^9*(35*a^4*b^3 + 42*a^5*b*c) + x^33*(35*b^3*c^4 + 42
*a*b*c^5) + x^12*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^18*(7*a*b^6 + 35*a^4*c^3 + 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c + 21*a^5*b^2) + x^36*(7*a*c^6 +
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

sympy [B] time = 11.76, size = 360, normalized size = 20.00

$$-\frac{1}{21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\left(147ac^6+441b^2c^5\right)+x^{33}\left(882abc^5+735b^3c^4\right)+x^{30}\left(441a^2c^5+2205ab^2c^4+735b^4c^3\right)+x^{27}\left(2205a^2bc^4+2940ab^3c^3+441b^5c^2\right)+x^{24}\left(735a^3c^4+4410a^2b^2c^3+2205ab^4c^2+147b^6c\right)+x^{21}\left(2940a^3bc^3+4410a^2b^3c^2+882ab^5c+21b^7\right)+x^{18}\left(735a^4c^3+4410a^3b^2c^2+2205a^2b^4c+147ab^6\right)+x^{15}\left(2205a^4bc^2+2940a^3b^3c+441a^2b^5\right)+x^{12}\left(441a^5c^2+2205a^4b^2c+735a^3b^4\right)+x^9\left(882a^5bc+735a^4b^3\right)+x^6\left(147a^6c+441a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] -1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(1
47*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30*(
441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4
 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2*b**
2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 4410*a*
*2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*a**3*b
**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2 + 2940*
a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**2*c + 73
5*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c + 441
*a**5*b**2))
```

3.90 $\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(a+bx^n+cx^{2n}\right)^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7n\left(a+x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*(a + x^n*(b + c*x^n))^7)
```

IntegrateAlgebraic [A] time = 0.07, size = 23, normalized size = 1.00

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*(a + b*x^n + c*x^(2*n))^7)
```

fricas [B] time = 1.06, size = 394, normalized size = 17.13

$$-\frac{1}{7\left(c^{7} n x^{14 n} + 7 b c^{6} n x^{13 n} + 7 a^{6} b n x^{n} + a^{7} n + 7\left(3 b^{2} c^{5} + a c^{6}\right) n x^{12 n} + 7\left(5 b^{3} c^{4} + 6 a b c^{5}\right) n x^{11 n} + 7\left(5 b^{4} c^{3} + 15 a b^{2} c^{4} + 3 a^{2} c^{5}\right) n x^{10 n} + 7\left(3 b^{5} c^{2} + 20 a b^{3} c^{3} + 15 a^{2} b c^{4}\right) n x^{9 n} + 7\left(b^{6} c + 15 a b^{4} c^{2} + 30 a^{2} b^{2} c^{3} + 5 a^{3} c^{4}\right) n x^{8 n} + \left(b^{7} + 42 a b^{5} c + 210 a^{2} b^{3} c^{2} + 140 a^{3} b c^{3}\right) n x^{7 n} + 7\left(a b^{6} + 15 a^{2} b^{4} c + 30 a^{3} b^{2} c^{2} + 5 a^{4} c^{3}\right) n x^{6 n} + 7\left(3 a^{2} b^{5} + 20 a^{3} b^{3} c + 15 a^{4} b c^{2}\right) n x^{5 n} + 7\left(5 a^{3} b^{4} + 15 a^{4} b^{2} c + 3 a^{5} c^{2}\right) n x^{4 n} + 7\left(5 a^{4} b^{3} + 6 a^{5} b c\right) n x^{3 n} + 7\left(3 a^{5} b^{2} + a^{6} c\right) n x^{2 n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3
*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^
(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3
*a^5*b^2 + a^6*c)*n*x^(2*n))
```

giac [A] time = 0.63, size = 21, normalized size = 0.91

$$-\frac{1}{7\left(cx^{2n} + bx^{n} + a\right)^{7} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

maple [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7\left(b\,x^{n} + c\,x^{2n} + a\right)^{7} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)+a)^8,x)
```

```
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

maxima [B] time = 2.39, size = 416, normalized size = 18.09

$$-\frac{1}{7\left(c^{7} n x^{14 n} + 7 b c^{6} n x^{13 n} + 7 a^{6} b n x^{n} + a^{7} n + 7\left(3 b^{2} c^{5} n + a c^{6} n\right) x^{12 n} + 7\left(5 b^{3} c^{4} n + 6 a b c^{5} n\right) x^{11 n} + 7\left(5 b^{4} c^{3} n + 15 a b^{2} c^{4} n + 3 a^{2} c^{5} n\right) x^{10 n} + 7\left(3 b^{5} c^{2} n + 20 a b^{3} c^{3} n + 15 a^{2} b c^{4} n\right) x^{9 n} + 7\left(b^{6} c n + 15 a b^{4} c^{2} n + 30 a^{2} b^{2} c^{3} n + 5 a^{3} c^{4} n\right) x^{8 n} + \left(b^{7} n + 42 a b^{5} c n + 210 a^{2} b^{3} c^{2} n + 140 a^{3} b c^{3} n\right) x^{7 n} + 7\left(a b^{6} n + 15 a^{2} b^{4} c n + 30 a^{3} b^{2} c^{2} n + 5 a^{4} c^{3} n\right) x^{6 n} + 7\left(3 a^{2} b^{5} n + 20 a^{3} b^{3} c n + 15 a^{4} b c^{2} n\right) x^{5 n} + 7\left(5 a^{3} b^{4} n + 15 a^{4} b^{2} c n + 3 a^{5} c^{2} n\right) x^{4 n} + 7\left(5 a^{4} b^{3} n + 6 a^{5} b c n\right) x^{3 n} + 7\left(3 a^{5} b^{2} n + a^{6} c n\right) x^{2 n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2*n))
```

mupad [B] time = 23.01, size = 496, normalized size = 21.57

$$-\frac{1}{7a^7n+7b^7nx^{7n}+7c^7nx^{14n}+49a^6bnx^n+49ab^6nx^{6n}+49a^6cnx^{2n}+49ac^6nx^{12n}+49b^6cnx^{8n}+49bc^6nx^{13n}+147a^5b^2nx^{2n}+245a^4b^3nx^{3n}+245a^3b^4nx^{4n}+147a^2b^5nx^{5n}+147a^5c^2nx^{4n}+245a^4c^3nx^{6n}+245a^3c^4nx^{8n}+147a^2c^5nx^{10n}+147b^5c^2nx^{9n}+245b^4c^3nx^{10n}+245b^3c^4nx^{11n}+147b^2c^5nx^{12n}+735a^4b^2cnx^{4n}+980a^3b^3cnx^{5n}+735a^4bc^2nx^{5n}+735a^2b^4cnx^{6n}+980a^3bc^3nx^{7n}+735ab^4c^2nx^{8n}+980ab^3c^3nx^{9n}+735a^2bc^4nx^{9n}+735ab^2c^4nx^{10n}+1470a^3b^2c^2nx^{6n}+1470a^2b^3c^2nx^{7n}+1470a^2b^2c^3nx^{8n}+294a^5bcnx^{3n}+294ab^5cnx^{7n}+294abc^5nx^{11n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*a^7*n + 7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n + 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) + 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) + 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) + 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) + 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) + 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) + 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) + 980*a^3*b*c^3*n*x^(7*n) + 735
*a*b^4*c^2*n*x^(8*n) + 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) +
735*a*b^2*c^4*n*x^(10*n) + 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) + 294*a^5*b*c*n*x^(3*n) + 294*a*b^5*c*
n*x^(7*n) + 294*a*b*c^5*n*x^(11*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{b+2cx}{-a+bx+cx^2} \, dx$

Optimal. Leaf size=13

$$\log\left(a-bx-cx^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {628}

$$\log\left(a-bx-cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[a - b*x - c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{-a+bx+cx^2} \, dx = \log\left(a-bx-cx^2\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.92

$$\log(x(b+cx)-a)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[-a + x*(b + c*x)]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{-a+bx+cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(-a + b*x + c*x^2), x]
```

fricas [A] time = 0.85, size = 13, normalized size = 1.00

$$\log\left(cx^2+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x - a)
```

giac [**A**] time = 0.39, size = 14, normalized size = 1.08

$$\log\left(\left|cx^2 + bx - a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x - a))
```

maple [**A**] time = 0.00, size = 14, normalized size = 1.08

$$\ln\left(c\,x^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a),x)
[Out] ln(c*x^2+b*x-a)
```

maxima [**A**] time = 0.44, size = 13, normalized size = 1.00

$$\log\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x - a)
```

mupad [**B**] time = 0.05, size = 13, normalized size = 1.00

$$\ln\left(c\,x^2 + b\,x - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2),x)
[Out] log(b*x - a + c*x^2)
```

sympy [**A**] time = 0.15, size = 10, normalized size = 0.77

$$\log\left(-a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
[Out] log(-a + b*x + c*x**2)
```

3.92 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{-a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\log\left(a-bx^2-cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[-a + b*x^2 + c*x^4]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(b+2cx^2\right)}{-a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4), x]
```

fricas [A] time = 0.63, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(cx^4 + bx^2 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

giac [A] time = 1.62, size = 18, normalized size = 0.95

$$\frac{1}{2}\log\left(\left|cx^4 + bx^2 - a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2 - a))
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln\left(c\,x^4 + b\,x^2 - a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x)
[Out] 1/2*ln(c*x^4+b*x^2-a)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(cx^4 + bx^2 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^4 + b\,x^2 - a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4),x)
[Out] log(b*x^2 - a + c*x^4)/2
```

sympy [A] time = 0.28, size = 14, normalized size = 0.74

$$\frac{\log\left(-a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
[Out] log(-a + b*x**2 + c*x**4)/2
```

3.93 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{-a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\log\left(a-bx^3-cx^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[-a + b*x^3 + c*x^6]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b+2cx^3\right)}{-a+bx^3+cx^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6), x]
```

fricas [A] time = 0.76, size = 17, normalized size = 0.89

$$\frac{1}{3}\log\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

giac [A] time = 1.04, size = 18, normalized size = 0.95

$$\frac{1}{3}\log\left(\left|cx^6 + bx^3 - a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 - a))
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln\left(c\,x^6 + b\,x^3 - a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x)
[Out] 1/3*ln(c*x^6+b*x^3-a)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{3}\log\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^6 + b\,x^3 - a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)
[Out] log(b*x^3 - a + c*x^6)/3
```

sympy [A] time = 0.39, size = 14, normalized size = 0.74

$$\frac{\log\left(-a + bx^3 + cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
[Out] log(-a + b*x**3 + c*x**6)/3
```

3.94 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=21

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 628}

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{-a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\log\left(a-bx^n-cx^{2n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

IntegrateAlgebraic [A] time = 0.08, size = 21, normalized size = 1.00

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

fricas [A] time = 1.22, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^n - a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

giac [A] time = 0.37, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^n - a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

maple [A] time = 0.02, size = 26, normalized size = 1.24

$$\frac{\ln\left(-b\,e^{n\ln(x)} - c\,e^{2n\ln(x)} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x)
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

maxima [A] time = 0.60, size = 25, normalized size = 1.19

$$\frac{\log\left(\frac{cx^{2n}+bx^n-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

mupad [B] time = 2.68, size = 199, normalized size = 9.48

$$\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\,x^n\right)\right)\,\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)+\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\,x^n\right)\right)\,\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2+4\,a\,c}}\right)}{n\,\sqrt{b^2+4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n)),x)
[Out] log((2*c*x^n)/n - (1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*
c*x^n))*(1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) + log((2*c*x^n)/n
 - (1/n - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*c*x^n))*(1/n -
(b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*x^n)/(4*a*
c + b^2)^(1/2)))/(n*(4*a*c + b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

3.95 $\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx$

Optimal. Leaf size=18

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx=\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - x*(b + c*x))^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(-a + b*x + c*x^2)^8, x]
```

fricas [B] time = 0.98, size = 354, normalized size = 19.67

$$-\frac{1}{7\left(c^7x^{14}+7bc^6x^{13}+7\left(3b^2c^5-ac^6\right)x^{12}+7\left(5b^3c^4-6abc^5\right)x^{11}+7\left(5b^4c^3-15ab^2c^4+3a^2c^5\right)x^{10}+7\left(3b^5c^2-20ab^3c^3+15a^2bc^4\right)x^9+7\left(b^6c-15ab^4c^2+30a^2b^2c^3-5a^3c^4\right)x^8+7a^6bx+\left(b^7-42ab^5c+210a^2b^3c^2-140a^3bc^3\right)x^7-a^7-7\left(ab^6-15a^2b^4c+30a^3b^2c^2-5a^4c^3\right)x^6+7\left(3a^2b^5-20a^3b^3c+15a^4bc^2\right)x^5-7\left(5a^3b^4-15a^4b^2c+3a^5c^2\right)x^4+7\left(5a^4b^3-6a^5bc\right)x^3-7\left(3a^5b^2-a^6c\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4 -
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2
- 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^
4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*(3*a^
5*b^2 - a^6*c)*x^2)
```

giac [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{7\left(c\,x^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x-a)^7
```

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

mupad [B] time = 5.22, size = 358, normalized size = 19.89

$$-\frac{1}{7\left(x^5\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\left(-140\,a^3\,b\,c^3+210\,a^2\,b^3\,c^2-42\,a\,b^5\,c+b^7\right)+x^3\left(35\,a^4\,b^3-42\,a^5\,b\,c\right)+x^{11}\left(35\,b^3\,c^4-42\,a\,b\,c^5\right)-x^4\left(21\,a^5\,c^2-105\,a^4\,b^2\,c+35\,a^3\,b^4\right)+x^{10}\left(21\,a^2\,c^5-105\,a\,b^2\,c^4+35\,b^4\,c^3\right)-a^7-x^6\left(-35\,a^4\,c^3+210\,a^3\,b^2\,c^2-105\,a^2\,b^4\,c+7\,a\,b^6\right)+x^8\left(-35\,a^3\,c^4+210\,a^2\,b^2\,c^3-105\,a\,b^4\,c^2+7\,b^6\,c\right)+c^7\,x^{14}+x^2\left(7\,a^6\,c-21\,a^5\,b^2\right)-x^{12}\left(7\,a\,c^6-21\,b^2\,c^5\right)+7\,b\,c^6\,x^{13}+7\,a^6\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 -
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*a*b
*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 - 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

sympy [**B**] time = 5.06, size = 359, normalized size = 19.94

$$-\frac{1}{-7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\left(-49ac^6 + 147b^2c^5\right) + x^{11}\left(-294abc^5 + 245b^3c^4\right) + x^{10}\left(147a^2c^5 - 735ab^2c^4 + 245b^4c^3\right) + x^9\left(735a^2bc^4 - 980ab^3c^3 + 147b^5c^2\right) + x^8\left(-245a^3c^4 + 1470a^2b^2c^3 - 735ab^4c^2 + 49b^6c\right) + x^7\left(-980a^3bc^3 + 1470a^2b^3c^2 - 294ab^5c + 7b^7\right) + x^6\left(245a^4c^3 - 1470a^3b^2c^2 + 735a^2b^4c - 49ab^6\right) + x^5\left(735a^4bc^2 - 980a^3b^3c + 147a^2b^5\right) + x^4\left(-147a^5c^2 + 735a^4b^2c - 245a^3b^4\right) + x^3\left(-294a^5bc + 245a^4b^3\right) + x^2\left(49a^6c - 147a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c
**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a
**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 -
735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735
*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a
**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*
(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))
```

3.96 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{\left(-a+bx^2+cx^4\right)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx,x,x^2\right)\\ &= \frac{1}{14\left(a-bx^2-cx^4\right)^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{14\left(-a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(-a + b*x^2 + c*x^4)^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(b+2cx^2\right)}{\left(-a+bx^2+cx^4\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8, x]
```

fricas [B] time = 1.00, size = 356, normalized size = 17.80

$$-\frac{1}{14\left(c^{7}x^{28}+7bc^{6}x^{26}+7\left(3b^{2}c^{5}-ac^{6}\right)x^{24}+7\left(5b^{3}c^{4}-6abc^{5}\right)x^{22}+7\left(5b^{4}c^{3}-15ab^{2}c^{4}+3a^{2}c^{5}\right)x^{20}+7\left(3b^{5}c^{2}-20ab^{3}c^{3}+15a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c-15ab^{4}c^{2}+30a^{2}b^{2}c^{3}-5a^{3}c^{4}\right)x^{16}+\left(b^{7}-42ab^{5}c+210a^{2}b^{3}c^{2}-140a^{3}bc^{3}\right)x^{14}-7\left(ab^{6}-15a^{2}b^{4}c+30a^{3}b^{2}c^{2}-5a^{4}c^{3}\right)x^{12}+7\left(3a^{2}b^{5}-20a^{3}b^{3}c+15a^{4}bc^{2}\right)x^{10}+7a^{6}bx^{2}-7\left(5a^{3}b^{4}-15a^{4}b^{2}c+3a^{5}c^{2}\right)x^{8}-a^{7}+7\left(5a^{4}b^{3}-6a^{5}bc\right)x^{6}-7\left(3a^{5}b^{2}-a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

giac [A] time = 7.19, size = 18, normalized size = 0.90

$$-\frac{1}{14\left(cx^{4}+bx^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 - a)^7
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2-a)^7
```

maxima [B] time = 0.97, size = 356, normalized size = 17.80

$$-\frac{1}{14\left(c^{7}x^{28}+7bc^{6}x^{26}+7\left(3b^{2}c^{5}-ac^{6}\right)x^{24}+7\left(5b^{3}c^{4}-6abc^{5}\right)x^{22}+7\left(5b^{4}c^{3}-15ab^{2}c^{4}+3a^{2}c^{5}\right)x^{20}+7\left(3b^{5}c^{2}-20ab^{3}c^{3}+15a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c-15ab^{4}c^{2}+30a^{2}b^{2}c^{3}-5a^{3}c^{4}\right)x^{16}+\left(b^{7}-42ab^{5}c+210a^{2}b^{3}c^{2}-140a^{3}bc^{3}\right)x^{14}-7\left(ab^{6}-15a^{2}b^{4}c+30a^{3}b^{2}c^{2}-5a^{4}c^{3}\right)x^{12}+7\left(3a^{2}b^{5}-20a^{3}b^{3}c+15a^{4}bc^{2}\right)x^{10}+7a^{6}bx^{2}-7\left(5a^{3}b^{4}-15a^{4}b^{2}c+3a^{5}c^{2}\right)x^{8}-a^{7}+7\left(5a^{4}b^{3}-6a^{5}bc\right)x^{6}-7\left(3a^{5}b^{2}-a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

mupad [B] time = 11.04, size = 360, normalized size = 18.00

$$-\frac{1}{14\left(x^{10}\left(105a^4bc^2-140a^3b^3c+21a^2b^5\right)+x^{18}\left(105a^2bc^4-140ab^3c^3+21b^5c^2\right)+x^{14}\left(-140a^3bc^3+210a^2b^3c^2-42ab^5c+b^7\right)+x^6\left(35a^4b^3-42a^5bc\right)+x^{22}\left(35b^3c^4-42abc^5\right)-x^8\left(21a^5c^2-105a^4b^2c+35a^3b^4\right)+x^{20}\left(21a^2c^5-105ab^2c^4+35b^4c^3\right)-a^7-x^{12}\left(-35a^4c^3+210a^3b^2c^2-105a^2b^4c+7ab^6\right)+x^{16}\left(-35a^3c^4+210a^2b^2c^3-105ab^4c^2+7b^6c\right)+c^7x^{28}+x^4\left(7a^6c-21a^5b^2\right)-x^{24}\left(7ac^6-21b^2c^5\right)+7a^6bx^2+7bc^6x^{26}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^6*(35*a^4*b^3 - 42*a^5*b*c) + x^22*(35*b^3*c^4 - 42
*a*b*c^5) - x^8*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^12*(7*a*b^6 - 35*a^4*c^3 - 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c - 21*a^5*b^2) - x^24*(7*a*c^6 - 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

sympy [B] time = 8.09, size = 360, normalized size = 18.00

$$-\frac{1}{-14a^7+98a^6bx^2+98bc^6x^{26}+14c^7x^{28}+x^{24}\left(-98ac^6+294b^2c^5\right)+x^{22}\left(-588abc^5+490b^3c^4\right)+x^{20}\left(294a^2c^5-1470ab^2c^4+490b^4c^3\right)+x^{18}\left(1470a^2bc^4-1960ab^3c^3+294b^5c^2\right)+x^{16}\left(-490a^3c^4+2940a^2b^2c^3-1470ab^4c^2+98b^6c\right)+x^{14}\left(-1960a^3bc^3+2940a^2b^3c^2-588ab^5c+14b^7\right)+x^{12}\left(490a^4c^3-2940a^3b^2c^2+1470a^2b^4c-98ab^6\right)+x^{10}\left(1470a^4bc^2-1960a^3b^3c+294a^2b^5\right)+x^8\left(-294a^5c^2+1470a^4b^2c-490a^3b^4\right)+x^6\left(-588a^5bc+490a^4b^3\right)+x^4\left(98a^6c-294a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] -1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-9
8*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20*(
294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4
 - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**2*b*
*2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2940*a
**2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940*a**3*
b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2 - 1960*
a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**2*c - 49
0*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c - 294
*a**5*b**2))
```

3.97 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{\left(-a+bx^3+cx^6\right)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx,x,x^3\right)\\ &= \frac{1}{21\left(a-bx^3-cx^6\right)^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{21\left(-a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(-a + b*x^3 + c*x^6)^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b+2cx^3\right)}{\left(-a+bx^3+cx^6\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8, x]
```

fricas [B] time = 0.76, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{27}+7\left(b^{6}c-15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}-5\,a^{3}c^{4}\right)x^{24}+\left(b^{7}-42\,ab^{5}c+210\,a^{2}b^{3}c^{2}-140\,a^{3}bc^{3}\right)x^{21}-7\left(ab^{6}-15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}-5\,a^{4}c^{3}\right)x^{18}+7\left(3\,a^{2}b^{5}-20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{15}-7\left(5\,a^{3}b^{4}-15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{12}+7\,a^{6}bx^{3}+7\left(5\,a^{4}b^{3}-6\,a^{5}bc\right)x^{9}-a^{7}-7\left(3\,a^{5}b^{2}-a^{6}c\right)x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

giac [A] time = 22.35, size = 18, normalized size = 0.90

$$-\frac{1}{21\left(cx^{6}+bx^{3}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 - a)^7
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3-a)^7
```

maxima [B] time = 0.94, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{27}+7\left(b^{6}c-15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}-5\,a^{3}c^{4}\right)x^{24}+\left(b^{7}-42\,ab^{5}c+210\,a^{2}b^{3}c^{2}-140\,a^{3}bc^{3}\right)x^{21}-7\left(ab^{6}-15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}-5\,a^{4}c^{3}\right)x^{18}+7\left(3\,a^{2}b^{5}-20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{15}-7\left(5\,a^{3}b^{4}-15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{12}+7\,a^{6}bx^{3}+7\left(5\,a^{4}b^{3}-6\,a^{5}bc\right)x^{9}-a^{7}-7\left(3\,a^{5}b^{2}-a^{6}c\right)x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

mupad [B] time = 16.60, size = 360, normalized size = 18.00

$$-\frac{1}{21\left(x^{15}\left(105a^4bc^2-140a^3b^3c+21a^2b^5\right)+x^{27}\left(105a^2bc^4-140ab^3c^3+21b^5c^2\right)+x^{21}\left(-140a^3bc^3+210a^2b^3c^2-42ab^5c+b^7\right)+x^9\left(35a^4b^3-42a^5bc\right)+x^{33}\left(35b^3c^4-42abc^5\right)-x^{12}\left(21a^5c^2-105a^4b^2c+35a^3b^4\right)+x^{30}\left(21a^2c^5-105ab^2c^4+35b^4c^3\right)-a^7-x^{18}\left(-35a^4c^3+210a^3b^2c^2-105a^2b^4c+7ab^6\right)+x^{24}\left(-35a^3c^4+210a^2b^2c^3-105ab^4c^2+7b^6c\right)+c^7x^{42}+x^6\left(7a^6c-21a^5b^2\right)-x^{36}\left(7ac^6-21b^2c^5\right)+7a^6bx^3+7bc^6x^{39}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b*c) + x^33*(35*b^3*c^4 - 42
*a*b*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^18*(7*a*b^6 - 35*a^4*c^3 - 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c - 21*a^5*b^2) - x^36*(7*a*c^6 -
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

sympy [B] time = 11.79, size = 360, normalized size = 18.00

$$-\frac{1}{-21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\left(-147ac^6+441b^2c^5\right)+x^{33}\left(-882abc^5+735b^3c^4\right)+x^{30}\left(441a^2c^5-2205ab^2c^4+735b^4c^3\right)+x^{27}\left(2205a^2bc^4-2940ab^3c^3+441b^5c^2\right)+x^{24}\left(-735a^3c^4+4410a^2b^2c^3-2205ab^4c^2+147b^6c\right)+x^{21}\left(-2940a^3bc^3+4410a^2b^3c^2-882ab^5c+21b^7\right)+x^{18}\left(735a^4c^3-4410a^3b^2c^2+2205a^2b^4c-147ab^6\right)+x^{15}\left(2205a^4bc^2-2940a^3b^3c+441a^2b^5\right)+x^{12}\left(-441a^5c^2+2205a^4b^2c-735a^3b^4\right)+x^9\left(-882a^5bc+735a^4b^3\right)+x^6\left(147a^6c-441a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] -1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(
-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x**3
0*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c
**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*a**2
*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3 + 44
10*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 - 4410*a
**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*c**2 -
2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a**4*b**2*
c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*
c - 441*a**5*b**2))
```

3.98 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(-a+bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(-a+bx^n+cx^{2n}\right)^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx, x, x^n\right)}{n}\\ &= \frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}\end{aligned}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 0.92

$$\frac{1}{7n\left(a-x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - x^n*(b + c*x^n))^7)
```

IntegrateAlgebraic [A] time = 0.08, size = 25, normalized size = 1.00

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

fricas [**B**] time = 0.97, size = 397, normalized size = 15.88

$$-\frac{1}{7\left(c^{7}nx^{14n}+7\,bc^{6}nx^{13n}+7\,a^{6}bnx^{n}-a^{7}n+7\left(3\,b^{2}c^{5}-ac^{6}\right)nx^{12n}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)nx^{11n}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)nx^{10n}+7\left(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)nx^{9n}+7\left(b^{6}c-15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}-5\,a^{3}c^{4}\right)nx^{8n}+\left(b^{7}-42\,ab^{5}c+210\,a^{2}b^{3}c^{2}-140\,a^{3}bc^{3}\right)nx^{7n}-7\left(ab^{6}-15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}-5\,a^{4}c^{3}\right)nx^{6n}+7\left(3\,a^{2}b^{5}-20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)nx^{5n}-7\left(5\,a^{3}b^{4}-15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)nx^{4n}+7\left(5\,a^{4}b^{3}-6\,a^{5}bc\right)nx^{3n}-7\left(3\,a^{5}b^{2}-a^{6}c\right)nx^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3
*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^
(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n) - 7*(3
*a^5*b^2 - a^6*c)*n*x^(2*n))
```

giac [**A**] time = 0.79, size = 23, normalized size = 0.92

$$-\frac{1}{7\left(cx^{2n}+bx^{n}-a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

maple [**A**] time = 0.07, size = 24, normalized size = 0.96

$$\frac{1}{7\left(-b\,x^{n}-c\,x^{2n}+a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)
```

```
[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

maxima [**B**] time = 2.42, size = 419, normalized size = 16.76

$$-\frac{1}{7\left(c^{7}nx^{14n}+7\,bc^{6}nx^{13n}+7\,a^{6}bnx^{n}-a^{7}n+7\left(3\,b^{2}c^{5}n-ac^{6}n\right)x^{12n}+7\left(5\,b^{3}c^{4}n-6\,abc^{5}n\right)x^{11n}+7\left(5\,b^{4}c^{3}n-15\,ab^{2}c^{4}n+3\,a^{2}c^{5}n\right)x^{10n}+7\left(3\,b^{5}c^{2}n-20\,ab^{3}c^{3}n+15\,a^{2}bc^{4}n\right)x^{9n}+7\left(b^{6}cn-15\,ab^{4}c^{2}n+30\,a^{2}b^{2}c^{3}n-5\,a^{3}c^{4}n\right)x^{8n}+\left(b^{7}n-42\,ab^{5}cn+210\,a^{2}b^{3}c^{2}n-140\,a^{3}bc^{3}n\right)x^{7n}-7\left(ab^{6}n-15\,a^{2}b^{4}cn+30\,a^{3}b^{2}c^{2}n-5\,a^{4}c^{3}n\right)x^{6n}+7\left(3\,a^{2}b^{5}n-20\,a^{3}b^{3}cn+15\,a^{4}bc^{2}n\right)x^{5n}-7\left(5\,a^{3}b^{4}n-15\,a^{4}b^{2}cn+3\,a^{5}c^{2}n\right)x^{4n}+7\left(5\,a^{4}b^{3}n-6\,a^{5}bcn\right)x^{3n}-7\left(3\,a^{5}b^{2}n-a^{6}cn\right)x^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*b*c^2
```

```
*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2*n))
```

mupad [B] time = 22.40, size = 496, normalized size = 19.84

$$-\frac{1}{7 b^7 n x^{7n} - 7 a^7 n + 7 c^7 n x^{14n} + 49 a^6 b n x^n - 49 a b^6 n x^{6n} + 49 a^6 c n x^{2n} - 49 a c^6 n x^{12n} + 49 b^6 c n x^{8n} + 49 b c^6 n x^{13n} - 147 a^5 b^2 n x^{2n} + 245 a^4 b^3 n x^{3n} - 245 a^3 b^4 n x^{4n} + 147 a^2 b^5 n x^{5n} - 147 a^5 c^2 n x^{4n} + 245 a^4 c^3 n x^{6n} - 245 a^3 c^4 n x^{8n} + 147 a^2 c^5 n x^{10n} + 147 b^5 c^2 n x^{9n} + 245 b^4 c^3 n x^{10n} + 245 b^3 c^4 n x^{11n} + 147 b^2 c^5 n x^{12n} + 735 a^4 b^2 c n x^{4n} - 980 a^3 b^3 c n x^{5n} + 735 a^4 b c^2 n x^{5n} + 735 a^2 b^4 c n x^{6n} - 980 a^3 b c^3 n x^{7n} - 735 a b^4 c^2 n x^{8n} - 980 a b^3 c^3 n x^{9n} + 735 a^2 b c^4 n x^{9n} - 735 a b^2 c^4 n x^{10n} - 1470 a^3 b^2 c^2 n x^{6n} + 1470 a^2 b^3 c^2 n x^{7n} + 1470 a^2 b^2 c^3 n x^{8n} - 294 a^5 b c n x^{3n} - 294 a b^5 c n x^{7n} - 294 a b c^5 n x^{11n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) - 7*a^7*n + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n - 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) - 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) - 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) - 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) - 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) - 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) - 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) - 980*a^3*b*c^3*n*x^(7*n) - 735
*a*b^4*c^2*n*x^(8*n) - 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) -
735*a*b^2*c^4*n*x^(10*n) - 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) - 294*a^5*b*c*n*x^(3*n) - 294*a*b^5*c*
n*x^(7*n) - 294*a*b*c^5*n*x^(11*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.99 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{bx+cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2), x]
```

fricas [A] time = 0.81, size = 10, normalized size = 1.00

$$\log \left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

giac [A] time = 0.46, size = 11, normalized size = 1.10

$$\log\left(\left|cx^2 + bx\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln\left(\left(cx + b\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
[Out] ln(x*(c*x+b))
```

maxima [A] time = 0.44, size = 10, normalized size = 1.00

$$\log\left(cx^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\ \left(b + c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
[Out] log(x*(b + c*x))
```

sympy [A] time = 0.12, size = 8, normalized size = 0.80

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

3.100 $\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=16

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(b+2cx^2\right)}{bx^2+cx^4}\,dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^2\right)\\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4), x]
```

fricas [A] time = 0.61, size = 13, normalized size = 0.81

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

giac [A] time = 0.49, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(\left|cx^4+bx^2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2))
```

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x)+\frac{\ln\left(c\,x^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.81

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

sympy [A] time = 0.19, size = 12, normalized size = 0.75

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

3.101 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} \, dx$

Optimal. Leaf size=16

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1584, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(b+2cx^3\right)}{bx^3+cx^6}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (b + 2cx^3)}{bx^3 + cx^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6), x]
```

fricas [A] time = 0.87, size = 13, normalized size = 0.81

$$\frac{1}{3} \log \left(cx^3 + b\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

giac [A] time = 0.34, size = 15, normalized size = 0.94

$$\frac{1}{3} \log \left(\left|cx^6 + bx^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^6 + b*x^3))
```

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x) + \frac{\ln \left(c\,x^3 + b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{3} \log \left(cx^3 + b\right) + \frac{1}{3} \log \left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

mupad [B] time = 1.99, size = 13, normalized size = 0.81

$$\frac{\ln \left(c\,x^3 + b\right)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

sympy [**A**] time = 0.20, size = 12, normalized size = 0.75

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

3.102 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx &= \int \frac{b+2cx^n}{x(b+cx^n)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^n\right)}{n}\\
&= \log(x)+\frac{\log(b+cx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
[Out] Log[x] + Log[b + c*x^n]/n
```

IntegrateAlgebraic [A] time = 0.06, size = 24, normalized size = 1.60

$$\frac{\log(bn + cnx^n)}{n} + \frac{\log(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
[Out] Log[x^n]/n + Log[b*n + c*n*x^n]/n
```

fricas [A] time = 0.86, size = 17, normalized size = 1.13

$$\frac{n\log(x) + \log(cx^n + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

giac [A] time = 0.37, size = 17, normalized size = 1.13

$$\frac{\log(|cx^n + b|)}{n} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(c*x^n + b))/n + log(abs(x))
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x) + \frac{\ln\left(c\,e^{n\ln(x)} + b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

maxima [B] time = 0.44, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

mupad [B] time = 2.23, size = 28, normalized size = 1.87

$$\frac{2\left(\ln(b + c\,x^n) - \mathrm{atanh}\left(\frac{2\,c\,x^n}{b} + 1\right)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x)
```

```
[Out] (2*(log(b + c*x^n) - atanh((2*c*x^n)/b + 1)))/n
```

sympy [A] time = 31.23, size = 48, normalized size = 3.20

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \frac{n^2\log(x)}{n^2-n} - \frac{n\log(x)}{n^2-n} & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n), Eq(c, 0)), (log(x) + log
(b/c + x**n)/n, True))
```

3.103 $\int \frac{b+2cx}{(bx+cx^2)^8} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8, x]
```

fricas [B] time = 0.86, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

giac [A] time = 0.32, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7\left(cx+b\right)^7 b^7}+\frac{c^7}{\left(cx+b\right)^6 b^8}+\frac{4c^7}{\left(cx+b\right)^5 b^9}+\frac{12c^7}{\left(cx+b\right)^4 b^{10}}+\frac{30c^7}{\left(cx+b\right)^3 b^{11}}+\frac{66c^7}{\left(cx+b\right)^2 b^{12}}+\frac{132c^7}{\left(cx+b\right) b^{13}}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}-\frac{1}{7b^7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4
-4/b^9*c^2/x^5+1/b^8*c/x^6+132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^
11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^
6+1/7*c^7/b^7/(c*x+b)^7
```

maxima [A] time = 0.42, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

mupad [B] time = 4.30, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,{\left(b+c\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

sympy [B] time = 0.88, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

3.104 $\int \frac{x\left(b+2cx^2\right)}{\left(bx^2+cx^4\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(b+2cx^2\right)}{\left(bx^2+cx^4\right)^8}\,dx &= \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^2\right)\\
&= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(b+2cx^2\right)}{\left(bx^2+cx^4\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] IntegrateAlgebraic[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8, x]
```

fricas [B] time = 0.88, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

giac [A] time = 0.45, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^6}{7\left(cx^2+b\right)^7c}-\frac{b^5}{\left(cx^2+b\right)^6c}-\frac{4b^4}{\left(cx^2+b\right)^5c}-\frac{12b^3}{\left(cx^2+b\right)^4c}-\frac{30b^2}{\left(cx^2+b\right)^3c}-\frac{66b}{\left(cx^2+b\right)^2c}-\frac{132}{\left(cx^2+b\right)c}\right)c^8}{2b^{13}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{1}{14b^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x)
```

```
[Out] -1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x
^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12-1/2*c^8/b^13*(-12*b^3/c/(c*x^2+b)^4-30*b^2
/c/(c*x^2+b)^3-132/c/(c*x^2+b)-b^5/c/(c*x^2+b)^6-4*b^4/c/(c*x^2+b)^5-66*b/c
/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^7)
```

maxima [B] time = 0.52, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

mupad [B] time = 2.33, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,{\left(c\,x^2+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

sympy [B] time = 1.38, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

3.105 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(b+2cx^3\right)}{\left(bx^3+cx^6\right)^8} dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8} dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^3\right)\\
&= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(b + 2cx^3\right)}{\left(bx^3 + cx^6\right)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8, x]
```

fricas [B] time = 0.71, size = 81, normalized size = 5.06

$$-\frac{1}{21 \left(c^7 x^{42} + 7 \, bc^6 x^{39} + 21 \, b^2 c^5 x^{36} + 35 \, b^3 c^4 x^{33} + 35 \, b^4 c^3 x^{30} + 21 \, b^5 c^2 x^{27} + 7 \, b^6 c x^{24} + b^7 x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

giac [A] time = 0.61, size = 15, normalized size = 0.94

$$-\frac{1}{21 \left(cx^6 + bx^3\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

maple [B] time = 0.01, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^6}{7\left(c x^3+b\right)^7 c}-\frac{b^5}{\left(c x^3+b\right)^6 c}-\frac{4b^4}{\left(c x^3+b\right)^5 c}-\frac{12b^3}{\left(c x^3+b\right)^4 c}-\frac{30b^2}{\left(c x^3+b\right)^3 c}-\frac{66b}{\left(c x^3+b\right)^2 c}-\frac{132}{\left(c x^3+b\right) c}\right) c^8}{3b^{13}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}-\frac{10c^4}{b^{11}x^9}+\frac{4c^3}{b^{10}x^{12}}-\frac{4c^2}{3b^9x^{15}}+\frac{c}{3b^8x^{18}}-\frac{1}{21b^7x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x)
```

```
[Out] -1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x
^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18-1/3*c^8/b^13*(-12*b^3/c/(c*x^3+b)^4-30*
b^2/c/(c*x^3+b)^3-132/c/(c*x^3+b)-b^5/c/(c*x^3+b)^6-4*b^4/c/(c*x^3+b)^5-66*
b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^7)
```

maxima [B] time = 0.51, size = 81, normalized size = 5.06

$$-\frac{1}{21 \left(c^7 x^{42} + 7 \, bc^6 x^{39} + 21 \, b^2 c^5 x^{36} + 35 \, b^3 c^4 x^{33} + 35 \, b^4 c^3 x^{30} + 21 \, b^5 c^2 x^{27} + 7 \, b^6 c x^{24} + b^7 x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

mupad [B] time = 5.07, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,{\left(c\,x^3+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

sympy [B] time = 1.90, size = 87, normalized size = 5.44

$$-\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

3.106 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}\left(b+2cx^n\right)}{\left(bx^n+cx^{2n}\right)^8}\,dx &= \int \frac{x^{-1-7n}\left(b+2cx^n\right)}{\left(b+cx^n\right)^8}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}\end{aligned}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

IntegrateAlgebraic [A] time = 0.11, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

fricas [B] time = 0.91, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+21\,b^2c^5nx^{12n}+35\,b^3c^4nx^{11n}+35\,b^4c^3nx^{10n}+21\,b^5c^2nx^{9n}+7\,b^6cnx^{8n}+b^7nx^{7n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

giac [A] time = 0.44, size = 20, normalized size = 0.95

$$-\frac{1}{7\left(cx^{2n}+bx^n\right)^7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

maple [B] time = 0.05, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n}+\frac{c\,x^{-6n}}{b^8n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{66c^5x^{-2n}}{b^{12}n}+\frac{\left(9009b^5c\,x^n+20020b^4c^2x^{2n}+24024b^3c^3x^{3n}+16380b^2c^4x^{4n}+6006b\,c^5x^{5n}+924c^6x^{6n}+1716b^6\right)c^7}{7\left(c\,x^n+b\right)^7b^{13}n}-\frac{132c^6x^{-n}}{b^{13}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

maxima [B] time = 0.66, size = 612, normalized size = 29.14

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13n}+2342340\,bc^{12}x^{12n}+6426420\,b^2c^{11}x^{11n}+9579570\,b^3c^{10}x^{10n}+8270262\,b^4c^9x^{9n}+4018014\,b^5c^8x^{8n}+934362\,b^6c^7x^{7n}+45045\,b^7c^6x^{6n}-5005\,b^8c^5x^{5n}+1001\,b^9c^4x^{4n}-273\,b^{10}c^3x^{3n}+91\,b^{11}c^2x^{2n}-35\,b^{12}cx^n+15\,b^{13}}{b^{14}c^7nx^{14n}+7\,b^{15}c^6nx^{13n}+21\,b^{16}c^5nx^{12n}+35\,b^{17}c^4nx^{11n}+35\,b^{18}c^3nx^{10n}+21\,b^{19}c^2nx^{9n}+7\,b^{20}cnx^{8n}+b^{21}nx^{7n}}+\frac{360360\,c^7\log\left(x\right)}{b^{15}}-\frac{360360\,c^7\log\left(\frac{cx^n+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12n}+2342340\,bc^{11}x^{11n}+6426420\,b^2c^{10}x^{10n}+9579570\,b^3c^9x^{9n}+8270262\,b^4c^8x^{8n}+4018014\,b^5c^7x^{7n}+934362\,b^6c^6x^{6n}+45045\,b^7c^5x^{5n}-5005\,b^8c^4x^{4n}+1001\,b^9c^3x^{3n}-273\,b^{10}c^2x^{2n}+91\,b^{11}cx^n-35\,b^{12}}{b^{13}c^7nx^{13n}+7\,b^{14}c^6nx^{12n}+21\,b^{15}c^5nx^{11n}+35\,b^{16}c^4nx^{10n}+35\,b^{17}c^3nx^{9n}+21\,b^{18}c^2nx^{8n}+7\,b^{19}cnx^{7n}+b^{20}nx^{6n}}+\frac{360360\,c^6\log\left(x\right)}{b^{14}}-\frac{360360\,c^6\log\left(\frac{cx^n+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
```

```
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

mupad [B] time = 2.36, size = 107, normalized size = 5.10

$$-\frac{1}{7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,b^6\,c\,n\,x^{8\,n}+49\,b\,c^6\,n\,x^{13\,n}+147\,b^5\,c^2\,n\,x^{9\,n}+245\,b^4\,c^3\,n\,x^{10\,n}+245\,b^3\,c^4\,n\,x^{11\,n}+147\,b^2\,c^5\,n\,x^{12\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^
(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(
11*n) + 147*b^2*c^5*n*x^(12*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

3.107 $\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{(a+x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x)*(a + b*x + c*x^2)^p, x]
```

fricas [A] time = 0.79, size = 28, normalized size = 1.40

$$\frac{\left(cx^2+bx+a\right)\left(cx^2+bx+a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

giac [A] time = 0.41, size = 20, normalized size = 1.00

$$\frac{\left(cx^2+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{\left(c\,x^2+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x)
[Out] (c*x^2+b*x+a)^(p+1)/(p+1)
```

maxima [A] time = 0.42, size = 20, normalized size = 1.00

$$\frac{\left(cx^2+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

mupad [B] time = 2.04, size = 39, normalized size = 1.95

$$\left(\frac{a}{p+1}+\frac{b\,x}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^p,x)
[Out] (a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p
```

sympy [B] time = 57.11, size = 104, normalized size = 5.20

$$\begin{cases} \frac{a\left(a+bx+cx^2\right)^p}{p+1}+\frac{bx\left(a+bx+cx^2\right)^p}{p+1}+\frac{cx^2\left(a+bx+cx^2\right)^p}{p+1} & \text{for}\ p\neq -1 \\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{-4ac+b^2}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
[Out] Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p +
 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
True))
```

3.108 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1247, 629\}$

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx, x, x^2\right)\\ &= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
[Out] Defer[IntegrateAlgebraic][x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p, x]
```

fricas [A] time = 0.78, size = 33, normalized size = 1.32

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

giac [A] time = 0.46, size = 23, normalized size = 0.92

$$\frac{\left(cx^4 + bx^2 + a\right)^{p+1}}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(c\,x^4 + b\,x^2 + a\right)^{p+1}}{2p + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x)
[Out] 1/2*(c*x^4+b*x^2+a)^(p+1)/(p+1)
```

maxima [A] time = 0.60, size = 33, normalized size = 1.32

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

mupad [B] time = 2.09, size = 49, normalized size = 1.96

$$\left(c\,x^4 + b\,x^2 + a\right)^p\,\left(\frac{a}{2\,p + 2} + \frac{b\,x^2}{2\,p + 2} + \frac{c\,x^4}{2\,p + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)
[Out] (a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.109 $\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx$

Optimal. Leaf size=25

$$\frac{\left(a+b x^3+c x^6\right)^{p+1}}{3 (p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$\frac{\left(a+b x^3+c x^6\right)^{p+1}}{3 (p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx &= \frac{1}{3} \text{Subst}\left(\int (b+2 c x) \left(a+b x+c x^2\right)^p \, dx,x,x^3\right)\\ &= \frac{\left(a+b x^3+c x^6\right)^{1+p}}{3 (1+p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\left(a+b x^3+c x^6\right)^{p+1}}{3 (p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
[Out] Defer[IntegrateAlgebraic][x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p, x]
```

fricas [A] time = 0.81, size = 33, normalized size = 1.32

$$\frac{\left(cx^6 + bx^3 + a\right)\left(cx^6 + bx^3 + a\right)^p}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

giac [A] time = 0.32, size = 23, normalized size = 0.92

$$\frac{\left(cx^6 + bx^3 + a\right)^{p+1}}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{\left(c\,x^6 + b\,x^3 + a\right)^{p+1}}{3p + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x)
[Out] 1/3*(c*x^6+b*x^3+a)^(p+1)/(p+1)
```

maxima [A] time = 0.58, size = 33, normalized size = 1.32

$$\frac{\left(cx^6 + bx^3 + a\right)\left(cx^6 + bx^3 + a\right)^p}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

mupad [B] time = 2.12, size = 49, normalized size = 1.96

$$\left(c\,x^6 + b\,x^3 + a\right)^p\,\left(\frac{a}{3\,p + 3} + \frac{b\,x^3}{3\,p + 3} + \frac{c\,x^6}{3\,p + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)
[Out] (a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.110 $\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\left(a+x^n\left(b+cx^n\right)\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^
p, x]
```

fricas [A] time = 0.96, size = 38, normalized size = 1.41

$$\frac{\left(cx^{2\,n}+bx^{n}+a\right)\left(cx^{2\,n}+bx^{n}+a\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

giac [A] time = 0.84, size = 27, normalized size = 1.00

$$\frac{\left(cx^{2\,n}+bx^{n}+a\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

maple [A] time = 0.06, size = 40, normalized size = 1.48

$$\frac{\left(b\,x^{n}+c\,x^{2n}+a\right)\left(b\,x^{n}+c\,x^{2n}+a\right)^{p}}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n)+a)^p,x)
```

```
[Out] (a+b*x^n+c*(x^n)^2)/(p+1)/n*(a+b*x^n+c*(x^n)^2)^p
```

maxima [A] time = 0.72, size = 39, normalized size = 1.44

$$\frac{\left(cx^{2\,n}+bx^{n}+a\right)\left(cx^{2\,n}+bx^{n}+a\right)^{p}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

mupad [B] time = 2.57, size = 56, normalized size = 2.07

$$\left(a+b\,x^{n}+c\,x^{2\,n}\right)^{p}\,\left(\frac{a}{n\,\left(p+1\right)}+\frac{b\,x^{n}}{n\,\left(p+1\right)}+\frac{c\,x^{2\,n}}{n\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] (a + b*x^n + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*x^n)/(n*(p + 1)) + (c*x^(2*n)
)/(n*(p + 1)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.111 $\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=22

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(x(b+cx)-a)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x)*(-a + b*x + c*x^2)^p, x]
```

fricas [A] time = 0.82, size = 32, normalized size = 1.45

$$\frac{\left(cx^2+bx-a\right)\left(cx^2+bx-a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

giac [A] time = 0.40, size = 22, normalized size = 1.00

$$\frac{\left(cx^2+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(c\,x^2+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x)
```

```
[Out] (c*x^2+b*x-a)^(p+1)/(p+1)
```

maxima [A] time = 0.42, size = 22, normalized size = 1.00

$$\frac{\left(cx^2+bx-a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

mupad [B] time = 2.05, size = 42, normalized size = 1.91

$$\left(\frac{b\,x}{p+1}-\frac{a}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x-a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)
```

```
[Out] ((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p
```

sympy [B] time = 56.66, size = 104, normalized size = 4.73

$$\begin{cases} -\frac{a\left(-a+bx+cx^2\right)^p}{p+1}+\frac{bx\left(-a+bx+cx^2\right)^p}{p+1}+\frac{cx^2\left(-a+bx+cx^2\right)^p}{p+1} & \text{for}\ p\neq-1\\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{4ac+b^2}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(
p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) +
x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c))
, True))
```

3.112 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
[Out] Defer[IntegrateAlgebraic][x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p, x]
```

fricas [A] time = 0.83, size = 37, normalized size = 1.37

$$\frac{\left(cx^4 + bx^2 - a\right)\left(cx^4 + bx^2 - a\right)^p}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

giac [A] time = 0.45, size = 25, normalized size = 0.93

$$\frac{\left(cx^4 + bx^2 - a\right)^{p+1}}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{\left(c\,x^4 + b\,x^2 - a\right)^{p+1}}{2p + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x)
[Out] 1/2*(c*x^4+b*x^2-a)^(p+1)/(p+1)
```

maxima [A] time = 0.60, size = 37, normalized size = 1.37

$$\frac{\left(cx^4 + bx^2 - a\right)\left(cx^4 + bx^2 - a\right)^p}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

mupad [B] time = 2.05, size = 52, normalized size = 1.93

$$\left(c\,x^4 + b\,x^2 - a\right)^p\,\left(\frac{b\,x^2}{2\,p + 2} - \frac{a}{2\,p + 2} + \frac{c\,x^4}{2\,p + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)
[Out] (b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] Timed out
```

3.113 $\int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^p \, dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p \, dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p \, dx,x,x^3\right)\\ &= \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2 \left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
[Out] Defer[IntegrateAlgebraic][x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p, x]
```

fricas [A] time = 0.88, size = 37, normalized size = 1.37

$$\frac{\left(cx^6 + bx^3 - a\right)\left(cx^6 + bx^3 - a\right)^p}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

giac [A] time = 0.39, size = 25, normalized size = 0.93

$$\frac{\left(cx^6 + bx^3 - a\right)^{p+1}}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)
```

maple [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{\left(c\,x^6 + b\,x^3 - a\right)^{p+1}}{3p + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x)
[Out] 1/3*(c*x^6+b*x^3-a)^(p+1)/(p+1)
```

maxima [A] time = 0.59, size = 37, normalized size = 1.37

$$\frac{\left(cx^6 + bx^3 - a\right)\left(cx^6 + bx^3 - a\right)^p}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

mupad [B] time = 2.08, size = 52, normalized size = 1.93

$$\left(c\,x^6 + b\,x^3 - a\right)^p\,\left(\frac{b\,x^3}{3\,p + 3} - \frac{a}{3\,p + 3} + \frac{c\,x^6}{3\,p + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)
[Out] (b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
```

```
[Out] Timed out
```

3.114 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int(b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.97

$$\frac{(x^n(b+cx^n)-a)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))
^p, x]
```

fricas [A] time = 0.88, size = 42, normalized size = 1.45

$$\frac{\left(cx^{2n}+bx^{n}-a\right)\left(cx^{2n}+bx^{n}-a\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

giac [A] time = 0.82, size = 29, normalized size = 1.00

$$\frac{\left(cx^{2n}+bx^{n}-a\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

maple [A] time = 0.06, size = 45, normalized size = 1.55

$$-\frac{\left(-b\,x^{n}-c\,x^{2n}+a\right)\left(b\,x^{n}+c\,x^{2n}-a\right)^{p}}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] -(-c*(x^n)^2-b*x^n+a)/(p+1)/n*(-a+b*x^n+c*(x^n)^2)^p
```

maxima [A] time = 0.71, size = 43, normalized size = 1.48

$$\frac{\left(cx^{2n}+bx^{n}-a\right)\left(cx^{2n}+bx^{n}-a\right)^{p}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

mupad [B] time = 2.54, size = 59, normalized size = 2.03

$$\left(\frac{b\,x^{n}}{n\,\left(p+1\right)}-\frac{a}{n\,\left(p+1\right)}+\frac{c\,x^{2\,n}}{n\,\left(p+1\right)}\right)\,{\left(b\,x^{n}-a+c\,x^{2\,n}\right)}^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^p,x)
```

```
[Out] ((b*x^n)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*x^n - a
+ c*x^(2*n))^p
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.115 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x)*(b*x + c*x^2)^p, x]
```

fricas [A] time = 1.00, size = 26, normalized size = 1.37

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

giac [A] time = 0.41, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx+b)\,x\left(c\,x^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] (c*x+b)*x/(p+1)*(c*x^2+b*x)^p
```

maxima [A] time = 0.43, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

mupad [B] time = 2.03, size = 23, normalized size = 1.21

$$\frac{x\left(c\,x^2+b\,x\right)^p\,(b+c\,x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

sympy [A] time = 0.66, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx\left(bx+cx^2\right)^p}{p+1}+\frac{cx^2\left(bx+cx^2\right)^p}{p+1} & \text{for } p\neq -1 \\ \log(x)+\log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.116 $\int x \left(b + 2cx^2\right) \left(bx^2 + cx^4\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^2 + cx^4\right)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1588}

$$\frac{\left(bx^2 + cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \left(b + 2cx^2\right) \left(bx^2 + cx^4\right)^p \, dx = \frac{\left(bx^2 + cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^2 \left(x^2 \left(b + cx^2\right)\right)^p \left(\frac{cx^2}{b} + 1\right)^{-p} \left(2c(p+1)x^2 \, {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right) + b(p+2) \, {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/
b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x \left(b + 2cx^2\right) \left(bx^2 + cx^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p, x]
```

fricas [A] time = 0.85, size = 31, normalized size = 1.29

$$\frac{\left(cx^4 + bx^2\right)\left(cx^4 + bx^2\right)^p}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

giac [A] time = 0.40, size = 22, normalized size = 0.92

$$\frac{\left(cx^4 + bx^2\right)^{p+1}}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(c\,x^2 + b\right)x^2\left(c\,x^4 + b\,x^2\right)^p}{2p + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x)
[Out] 1/2*(c*x^2+b)*x^2/(p+1)*(c*x^4+b*x^2)^p
```

maxima [A] time = 0.59, size = 35, normalized size = 1.46

$$\frac{\left(cx^4 + bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [B] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^2\,\left(c\,x^2 + b\right)\,\left(c\,x^4 + b\,x^2\right)^p}{2\,\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)
[Out] (x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))
```

sympy [B] time = 17.12, size = 85, normalized size = 3.54

$$\begin{cases} \frac{bx^2\left(bx^2+cx^4\right)^p}{2p+2} + \frac{cx^4\left(bx^2+cx^4\right)^p}{2p+2} & \text{for } p \neq -1 \\ \log\left(x\right) + \frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} + \frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)
**p/(2*p + 2), Ne(p, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/c) + x)/2 + log(
I*sqrt(b)*sqrt(1/c) + x)/2, True))
```

3.117 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx = \frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^3 \left(x^3 \left(b+cx^3\right)\right)^p \left(\frac{cx^3}{b}+1\right)^{-p} \left(2c(p+1)x^3 \, {}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2) \, {}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3)/
b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p, x]
```

fricas [**A**] time = 0.77, size = 31, normalized size = 1.29

$$\frac{\left(cx^6+bx^3\right)\left(cx^6+bx^3\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

giac [**A**] time = 0.67, size = 22, normalized size = 0.92

$$\frac{\left(cx^6+bx^3\right)^{p+1}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3)^(p + 1)/(p + 1)
```

maple [**A**] time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(c\,x^3+b\right)x^3\left(c\,x^6+b\,x^3\right)^p}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x)
[Out] 1/3*(c*x^3+b)*x^3/(p+1)*(c*x^6+b*x^3)^p
```

maxima [**A**] time = 0.59, size = 35, normalized size = 1.46

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

mupad [**B**] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^3\,\left(c\,x^3+b\right)\,\left(c\,x^6+b\,x^3\right)^p}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)
[Out] (x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
[Out] Timed out
```

3.118 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=26

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2034, 629}

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx &= \frac{\operatorname{Subst}\left(\int (b+2cx)\left(bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [C] time = 0.13, size = 111, normalized size = 4.27

$$\frac{x^{-np}\left(x^n\left(b+cx^n\right)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((x^n*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p,
2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^(n*p)*(1 + (c*x^n)/b)^p)
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x^{-1+n}\left(b+2cx^{n}\right)\left(bx^{n}+cx^{2n}\right)^{p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p, x
]
```

fricas [A] time = 0.88, size = 36, normalized size = 1.38

$$\frac{\left(cx^{2\,n}+bx^{n}\right)\left(cx^{2\,n}+bx^{n}\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

giac [A] time = 0.83, size = 26, normalized size = 1.00

$$\frac{\left(cx^{2\,n}+bx^{n}\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

maple [C] time = 0.11, size = 155, normalized size = 5.96

$$\frac{\left(c\,x^{n}+b\right)x^{n}\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)x^{n}\right)+i\pi\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)x^{n}\right)^{2}+i\pi\,\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)x^{n}\right)^{2}-i\pi\mathrm{csgn}\left(i\left(c\,x^{n}+b\right)x^{n}\right)^{3}+2\ln\left(x^{n}\right)+2\ln\left(c\,x^{n}+b\right)\right)p}{2}}}{\left(p+1\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] x^n*(c*x^n+b)/(p+1)/n*exp(1/2*p*(-I*Pi*csgn(I*x^n*(c*x^n+b))^3+I*Pi*csgn(I*
x^n*(c*x^n+b))^2*csgn(I*x^n)+I*Pi*csgn(I*x^n*(c*x^n+b))^2*csgn(I*(c*x^n+b))
-I*Pi*csgn(I*x^n*(c*x^n+b))*csgn(I*x^n)*csgn(I*(c*x^n+b))+2*ln(x^n)+2*ln(c*
x^n+b)))
```

maxima [A] time = 0.77, size = 40, normalized size = 1.54

$$\frac{\left(cx^{2\,n}+bx^{n}\right)e^{\left(p\log\left(cx^{n}+b\right)+p\log\left(x^{n}\right)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

mupad [**B**] time = 2.13, size = 34, normalized size = 1.31

$$\frac{x^n\ (b+c\,x^n)\ \left(b\,x^n+c\,x^{2\,n}\right)^p}{n\ \left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x)
[Out] (x^n*(b + c*x^n)*(b*x^n + c*x^(2*n))^p)/(n*(p + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

3.119 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx$

Optimal. Leaf size=47

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1594, 1468, 628}

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx &= \int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\left(c\sqrt[3]{d}-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}dx^{2/3}\right)x^{2/3}}\,dx\\
&= 3\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{c\sqrt[3]{d}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^2}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3
)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

IntegrateAlgebraic [A] time = 0.07, size = 47, normalized size = 1.00

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/
3)*d^(2/3)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

fricas [A] time = 0.76, size = 33, normalized size = 0.70

$$-\frac{3\,\log\left(dx^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+c^{\frac{2}{3}}d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
```

```
[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2
/3))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[%%%{%%{[%%%{1,[1]%%%},0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%},0]:[1
,0,0,%%%{-1,[1]%%%}]%%},[2]%%%}+%%%{%%%{%%{[%%%{-1,[1]%%%},0,0]:[1,0,0,%%%{
-1,[1]%%%}]%%},[1]%%%},[1]%%%}+%%%{%%{[%%%{%%%{1,[2]%%%},[0]%%%},0,0]:[1,0,
0,%%%{-1,[1]%%%}]%%},[0]%%%} / %%%{%%%{%%{[-1,0]:[1,0,0,%%%{-1,[1]%%%}]%%},
[2]%%%},[2]%%%}+%%%{%%{[%%%{%%{[1,0,0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%},0,
0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%}+%%%{%%{[%%%{%%%{-1,[1]%%%},[1]%%%},0]:
[1,0,0,%%%{-1,[1]%%%}]%%},[0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{3\ln\left(-c^{\frac{1}{3}}d\,x^{\frac{2}{3}}+c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}-c\,d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3
)*d*x^(4/3)),x)
```

```
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*x^(2/3)*d-c*d^(1/3))
```

maxima [A] time = 0.45, size = 34, normalized size = 0.72

$$-\frac{3\,\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

mupad [B] time = 2.46, size = 31, normalized size = 0.66

$$-\frac{3\,\ln\left(x^{2/3}+\frac{c^{2/3}}{d^{2/3}}-\frac{c^{1/3}\,x^{1/3}}{d^{1/3}}\right)}{c^{1/3}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x)
```

```
[Out] -(3*log(x^(2/3) + c^(2/3)/d^(2/3) - (c^(1/3)*x^(1/3))/d^(1/3)))/(c^(1/3)*d^
(2/3))
```

sympy [C] time = 6.32, size = 126, normalized size = 2.68

$$-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}-\frac{\sqrt{3}i\sqrt{c^{\frac{4}{3}}}\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}+\frac{\sqrt{3}i\sqrt{c^{\frac{4}{3}}}\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(
2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

```
[Out] -3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*I*sqrt(c**(4/3))*sqrt(d*
*(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3))
+ x**(1/3) + sqrt(3)*I*sqrt(c**(4/3))*sqrt(d**(4/3))/(2*c**(1/3)*d))/(c**(1
/3)*d**(2/3))
```

Chapter 4

Appendix

Local contents

4.1 Download section 618
4.2 Listing of Grading functions 618

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```